Section 2.3 Some Applications of First Order Equations

1. A gas tank holds 200 gallons of pure jet fuel. Now a person is pumping the liquid out of this tank at a rate of 2 gallons per second while replacing it at an equal rate with a mixture which contains 50% water. Assume that the mixture within the tank remains well-stirred throughout this process.

(a) Find an initial value problem to describe the amount of water, $Q(t)$, in this tank.

Comments: (1) The total amount of the mixture in the tank remains unchanged.

(2) $\frac{dQ}{dt}$ = rate in - rate out.

(b) Solve the problem from part (a).

(c) Find the limiting amount of water in the tank as $t \to \infty$.

(d) How long will it take for the amount of water to reach 99% of its limiting value?

2. A young person opened an account with initial deposit of M_0 dollars in a bank that pays interest at an annual rate r. Let $M(t)$ be the amount of money in this account after t years.

(a) If the interest is compounded **continuously**, find $M(t)$.

Comment: Continuous compounding means $\frac{dM}{dt} = rM$.

(b) Suppose that the interest is compounded n times per year instead of continuous compounding. Then find $M(t)$.

(c) What's the limiting value of $M(t)$ from part (b) as $n \to \infty$?

Comment: Recall that $\lim_{x \to \infty}(1 + \frac{1}{x})^x = e$.

(d) Now assume continuous compounding. Also, suppose that person deposits k dollars to the account each year. Find $M(t)$.