Section 2.2 Separable Equations

1. Consider the equation $2\frac{dy}{dx} = \frac{x^2}{y}, \;\; y < 0$.
(a) What's the order of this differential equation? Is it linear or nonlinear?
(b) Does the method based on integrating factor (from Sec 2.1) work for this equation?
(c) Find the general solution in explicit form.

2. Consider the equation $2\frac{dy}{dx} = x^2 y, \;\; y > 0$.
(a) Is this equation linear or nonlinear?
(b) Does the method from Sec 2.1 work for this equation?
(c) Can this equation be solved by the same technique used in problem 1(c)?

3. Now consider $2\frac{dy}{dx} = x^2 + y$. Answer the same questions as in 2(a)(b)(c).

Definition: An equation is **separable** if it can be written in the form

$$M(x) + N(y)\frac{dy}{dx} = 0$$

Comments: (1) In order to use the technique from 1(c), the differential equation has to be separable, but not necessarily linear.
(2) In order to apply the method from Section 2.1, the differential equation has to be linear, but not necessarily separable.

4. Find the general solution to the equation $\frac{dy}{dx} = \frac{y \cos x}{1+2y^2}$.
Comment: It is not always possible to write the solution in explicit form.

5. Consider the initial value problem
$\frac{dy}{dx} = \frac{2x+5}{2y}$, $y(0) = 2$.
(a) Find the solution in explicit form.
Comment: If the solution has more than one branches, choose the one which is compatible with the initial condition.

(b) Determine the interval in which the solution is defined.
Comment: To choose the interval, look at the initial condition again.