Please **read all instructions** carefully before beginning.

- Do not open this test booklet until you are directed to do so.
- You have 75 minutes to complete this exam.
- If you finish early, go back and check your work.
- The graders will only see the work on the **printed pages**. You may use other scratch paper, but the graders will not see anything written there.
- You may use a **calculator** for doing arithmetic, but you should not need one. All other materials and aids are strictly prohibited.
- For full credit you must **show your work** so that your reasoning is clear, unless otherwise indicated.
- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.
- Good luck!

---

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 75 minutes, without notes or distractions.
Problem 1.

Consider
\[
A = \begin{pmatrix} 0 & 1 & -1 & 0 \\ -2 & -1 & -1 & 2 \\ 2 & 3 & 5 & 4 \\ 6 & 3 & -3 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} -2 \\ -8 \\ 4 \\ 0 \end{pmatrix}.
\]

a) Carry out Gaussian reduction with maximal partial pivoting to find a \( PA = LU \) decomposition. You should obtain

\[
U = \begin{pmatrix} 6 & 3 & -3 & 0 \\ 0 & 2 & 6 & 4 \\ 0 & 0 & -4 & -2 \\ 0 & 0 & 0 & 3 \end{pmatrix}.
\]

Please write the row operations you performed.
b) Write the elementary matrices for the row operations you performed.

c) Solve the equations $Ly = Pb$ and $Ux = y$ to find a solution of $Ax = b$.

d) Briefly explain why step b) is faster than solving $Ax = b$ using Gaussian elimination on the augmented matrix $(A \ | \ b)$, once you have a $PA = LU$ decomposition.
Problem 2. [15 points]

a) Compute the inverse of \( \begin{pmatrix} 1 & -2 & 3 \\ -2 & 6 & -5 \\ 2 & 3 & 9 \end{pmatrix} \).

Be sure to write out any row operations you perform.

\[
\begin{pmatrix} 1 & -2 & 3 \\ -2 & 6 & -5 \\ 2 & 3 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} \ldots \end{pmatrix}
\]

b) For which value(s) of \( k \) is \( \begin{pmatrix} 1 & -2 & 3 \\ -2 & 6 & k \\ 2 & 3 & 9 \end{pmatrix} \) not invertible?

\[
k = \ldots
\]
Problem 3. 

Consider

\[ A = \begin{pmatrix} 1 & 3 & -2 & 0 \\ -2 & -6 & 6 & -2 \\ 2 & 6 & 3 & -7 \end{pmatrix} \quad b = \begin{pmatrix} 2 \\ -8 \\ -10 \end{pmatrix}. \]

a) Find the parametric vector form of the solution set of \( Ax = b \). Be sure to write out any row operations you perform.

\[ x = \begin{pmatrix} \ \\ \end{pmatrix} + \begin{pmatrix} \ \\ \end{pmatrix} \]

b) Write down two different solutions of \( Ax = b \). (Your answer will be two vectors with numbers in them.)

\[ x_1 = \begin{pmatrix} \ \\ \end{pmatrix} \quad x_2 = \begin{pmatrix} \ \\ \end{pmatrix} \]

c) Find a set of vectors spanning the solution set of \( Ax = 0 \) (for the same matrix \( A \) above).

\[ \text{(solution set)} = \text{Span} \left\{ \begin{pmatrix} \ \\ \end{pmatrix} \right\} \]

d) Let \( v = (-1, 1, 1, 1) \). Check that \( Av = 0 \), and write \( v \) as a linear combination of the spanning vectors you obtained in c).

[Hint: what values do the free variables have to take?]
Problem 4. [20 points]

For a certain 2 × 2 matrix $A$, the solution set of $Ax = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is drawn.

a) Draw the solution set of $Ax = 0$ and the solution set of $Ax = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ in the grid below. Be sure to label which is which.

b) $\text{rank}(A) =$

c) Draw the span of the columns of $A$. Be precise!
Problem 5. [15 points]

Find examples of matrices with the following properties. If no such matrix exists, write “no way, man,” or use your favorite colloquialism instead. You need not justify your answers.

a) A $3 \times 5$ matrix of rank 4, in RREF.

b) A $2 \times 2$ matrix $A$ such that the solution set of $Ax = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ is a line, and $Ax = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ has no solutions.

c) Three vectors in $\mathbb{R}^3$, no two of which are collinear (scalar multiples of each other), that span a plane.

d) A $4 \times 4$ matrix $A$ with a pivot in every row such that $A(1, 2, -1, 1) = 0$. 
Problem 6. [10 points]

Consider the span

\[ V = \text{Span}\left\{\begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix},\begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix},\begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}\right\}. \]

a) Show that \begin{pmatrix} -4 \\ -4 \\ -4 \end{pmatrix} is in \( V \).

b) Show that \begin{pmatrix} -4 \\ -4 \\ 4 \end{pmatrix} is not in \( V \).

c) Circle one: \( V \) is a point line plane space.