1. **Row Echelon Form**

For each of the following linear systems/augmented matrices, do the following:

(1) If it is a linear system, convert it to an augmented matrix. If it is an augmented matrix, convert it to a linear system.

(2) Decide whether or not the augmented matrix is in Row Echelon Form (REF).
   If it is in REF, circle the pivots/pivot entries. If it is not, explain why not.

a) \[ x + 2y = 1 \]
   \[ 3y = 2 \]

b) \[
\begin{pmatrix}
5 & -1 & 1 \\
0 & 0 & 6 \\
0 & 0 & 0
\end{pmatrix}
\]

c) \[ 2x + y - z = 1 \]
   \[ 3y + z = 2 \]
   \[ 2x = 1 \]

d) \[
\begin{pmatrix}
2 & 1 & 3 \\
0 & 1 & 5
\end{pmatrix}
\]

e) \[
\begin{pmatrix}
3 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 \\
-1 & -2 & 4 & 1
\end{pmatrix}
\]

f) \[
\begin{pmatrix}
5 & 5 & 5 \\
1 & 1 & 1
\end{pmatrix}
\]
2. **Two Equations and Two Unknowns**

Consider the system of 2 linear equations:

\[
\begin{align*}
x - y &= 2 \\
2x - 4y &= -4.
\end{align*}
\]

a) Draw the two lines in \( \mathbb{R}^2 \) determined by these two equations.

Now, solve the linear system using the following steps:

b) Use one row operation to eliminate \( x \) from the second equation.

c) Use another row operation to make the second equation into \( y = (?) \).

d) Use a third row operation to make the first equation into \( x = (?) \). What is the solution \((x, y)\)?

e) Check your answer by plugging your solution in to the original equations.

The first row operation is an elimination step, while the third is a substitution step.

f) After which row operation is the system in REF? Circle the pivot entries.
3. Three Equations Three Unknowns
Consider the system of three linear equations
\[
\begin{align*}
    x_1 - 3x_2 + x_3 &= 4 \\
    2x_1 - 8x_2 + 8x_3 &= -2 \\
    -6x_1 + 3x_2 - 15x_3 &= 9.
\end{align*}
\]

a) Convert this linear system into a matrix equation \(Ax = b\), where
\[
x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}
\]

b) Write down the augmented matrix \((A \mid b)\).

c) Use elementary row operations to convert the augmented matrix into a Row Echelon Form matrix.

\textbf{Hint:} Begin by eliminating \(x_1\): add a multiple of the first row to the second and third rows, so that the 2 and \(-6\) in the first column are replaced with 0. This takes two row operations.

d) How many elementary row operations did you use?

e) Convert the augmented matrix back into a system of three linear equations, and use back-substitution to find the solution vector \(x\).

f) Check your answer by multiplying \(A\) by \(x\) and confirming that it equals \(b\).

4. Another One—What’s Different?
Consider the system of three linear equations
\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= -2 \\
    2x_1 - 4x_2 + 8x_3 &= 2 \\
    x_1 - 3x_2 - x_3 &= 1.
\end{align*}
\]

a) Use row operations to eliminate \(x_1\) from the second and third equation.

b) You can now use a single row operation to put the linear system into REF. What row operation is it?

c) Substitute and solve for \((x_1, x_2, x_3)\). Check your answer with original linear system.
5. **Traffic Jam**

You should have seen this traffic example in lecture, although the numbers were a bit different. We'll now will explore it in more detail.

This represents a town with 4 main roads, as well as 8 roads in and out of town. Each road is one-way, in the direction indicated by the arrows. The 8 roads have a fixed number of cars/hour which travel in and out of town on them.

**Question:** How many cars/hour travel on each of the 4 main roads?

We'll use linear algebra to answer this question. At each intersection, the number of incoming cars per hour must equal the number of outgoing cars. This gives 4 linear equations, in the variables $x, y, z, w$:

\[
\begin{align*}
120 + w &= 250 + x \\
120 + x &= 70 + y \\
390 + y &= 250 + z \\
115 + z &= 175 + w.
\end{align*}
\]

a) Rewrite these equations with the variables on the left and the numbers on the right. Write the variables on the left side in neat columns.

b) Use the first equation to eliminate $x$ from the equations below it.

c) Use the second equation to eliminate $y$ from the equations below it.

d) Use the third equation to eliminate $z$ from the equations below it.

Remember: after each step, you should have a linear system with 4 equations and 4 variables.

e) At this point you are done with elimination. Can you substitute to solve? Can you answer the original **Question**?

f) How many solutions does this system have? Can you explain why from the original picture?
g) Convert the linear system obtained after step 3 into an augmented matrix. It should be in REF. Circle the pivot entries. Does each row have a pivot entry? Does each column to the left of the augmentation line have a pivot entry?