**Math 218D Problem Session**

Week 1

## 1. Row Echelon Form

For each of the following linear systems/augmented matrices, do the following:

(1) If it is a linear system, convert it to an augmented matrix. If it is an augmented matrix, convert it to a linear system.

(2) Decide whether or not the augmented matrix is in Row Echelon Form (REF). If it is in REF, circle the pivots/pivot entries. If it is not, explain why not.

**a)**

$$\begin{aligned} x + 2y &= 1 \\ 3y &= 2 \end{aligned}$$

**b)**

$$\left(\begin{array}{cc|c} 5 & -1 & 1 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{array}\right)$$

**c)**

$$\begin{aligned} 2x + \phantom{3}y - \phantom{3}z &= 1 \\ 3y + \phantom{3}z &= 2 \\ 2x &= 1 \end{aligned}$$

**d)**

$$\left(\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & 1 & 5 \end{array}\right)$$

**e)**

$$\left(\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -2 & 4 & 1 \end{array}\right)$$

**f)**

$$\left(\begin{array}{cc|c} 5 & 5 & 5 \\ 1 & 1 & 1 \end{array}\right)$$

## 2. Two Equations and Two Unknowns

Consider the system of 2 linear equations:

$$\begin{aligned} x - \phantom{4}y &= \phantom{-}2 \\ 2x - 4y &= -4. \end{aligned}$$

**a)** Draw the two lines in $\mathbf{R}^2$ determined by these two equations.

Now, solve the linear system using the following steps:

**b)** Use one row operation to eliminate $x$ from the second equation.

**c)** Use another row operation to make the second equation into $y = (?)$.

**d)** Use a third row operation to make the first equation into $x = (?)$. What is the solution $(x, y)$?

**e)** Check your answer by plugging your solution in to the original equations.

The first row operation is an *elimination* step, while the third is a *substitution* step.

**f)** After which row operation is the system in REF? Circle the pivot entries.

## 3. Three Equations Three Unknowns

Consider the system of three linear equations

$$\begin{aligned} x_1 - 3x_2 + \phantom{8}x_3 &= \phantom{-}4 \\ 2x_1 - 8x_2 + 8x_3 &= -2 \\ -6x_1 + 3x_2 - 15x_3 &= \phantom{-}9. \end{aligned}$$

**a)** Convert this linear system into a **matrix equation** $Ax = b$, where

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

**b)** Write down the augmented matrix $\left(A \mid b\right)$.

**c)** Use elementary row operations to convert the augmented matrix into a Row Echelon Form matrix.

**Hint:** Begin by eliminating $x_1$: add a multiple of the first row to the second and third rows, so that the 2 and $-6$ in the first column are replaced with 0. This takes two row operations.

**d)** How many elementary row operations did you use?

**e)** Convert the augmented matrix back into a system of three linear equations, and use back-substitution to find the solution vector $x$.

**f)** Check your answer by multiplying $A$ by $x$ and confirming that it equals $b$.

## 4. Another One—What's Different?

Consider the system of three linear equations

$$\begin{aligned} x_1 - 2x_2 + x_3 &= -2 \\ 2x_1 - 4x_2 + 8x_3 &= \phantom{-}2 \\ x_1 - 3x_2 - x_3 &= \phantom{-}1. \end{aligned}$$

**a)** Use row operations to eliminate $x_1$ from the second and third equation.

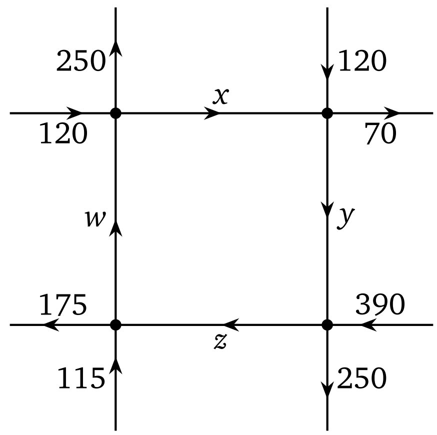
**b)** You can now use a single row operation to put the linear system in to REF. What row operation is it?

**c)** Substitute and solve for $(x_1, x_2, x_3)$. Check your answer with original linear system.

## 5. **Traffic Jam**

You should have seen this traffic example in lecture, although the numbers were a bit different. We'll now will explore it in more detail.

Traffic flow (cars/hr)




This represents a town with 4 main roads, as well as 8 roads in and out of town. Each road is one-way, in the direction indicated by the arrows. The 8 roads have a fixed number of cars/hour which travel in and out of town on them.

**Question:** How many cars/hour travel on each of the 4 main roads?

We'll use linear algebra to answer this question. At each intersection, the number of incoming cars per hour must equal the number of outgoing cars. This gives 4 linear equations, in the variables $x, y, z, w$:

$$\begin{aligned} 120 + w &= 250 + x \\ 120 + x &= 70 + y \\ 390 + y &= 250 + z \\ 115 + z &= 175 + w. \end{aligned}$$

**a)** Rewrite these equations with the variables on the left and the numbers on the right. Write the variables on the left side in neat columns.

**b)** Use the first equation to eliminate $x$ from the equations below it.

**c)** Use the second equation to eliminate $y$ from the equations below it.

**d)** Use the third equation to eliminate $z$ from the equations below it.

Remember: after each step, you should have a linear system with 4 equations and 4 variables.

**e)** At this point you are done with elimination. Can you substitute to solve? Can you answer the original **Question**?

**f)** How many solutions does this system have? Can you explain why from the original picture?

**g)** Convert the linear system obtained after step 3 into an augmented matrix. It should be in REF. Circle the pivot entries. Does each row have a pivot entry? Does each column to the left of the augmentation line have a pivot entry?