Welcome!

Linear Algebra
Deals with linear equations

- \( y = 3x + 2 \) \( \rightarrow \) \(-3x+y=2\)
  one eq in 2 variables

- \( \begin{cases} x+y+z=1 \\ y-2z=-3 \end{cases} \) \( \rightarrow \) two eqs
  in 3 variables

Equations of terms that look like
(const) \cdot (variable) \ or \ (const)

\( xy+z=1 \) \( \times \) \( x+3z=2 \) \( e^x = \cos(y) \)

Eg:

Q: How many cars are on the unlabeled roads?

120+w = 250+x \( \rightarrow -x+w = 130 \)

x+120 = 70+y \( \rightarrow x-y = -50 \)

y+390 = 530+z \( \rightarrow y-z = 140 \)

z+115 = 175+w \( \rightarrow z-w = 60 \)

4 equations in 4 unknowns!
An asteroid has been observed at coords 
(0,2), (2,1), (1,-1), (-1,-2), (-3,1), (-1,1)

Q: What is the orbit? Will it crash into Earth?

Orbit is an ellipse

Ellipses: 
\[ x^2 + By^2 + Cxy + Dx + Ey + F = 0 \]

(0,2): \[ 0 + 4B + 0 + 0 + 2E + F = 0 \]
(2,1): \[ 4 + B + 2C + 2D + E + F = 0 \]
(1,-1): \[ 1 + B - C + D - E + F = 0 \]

\[ \Rightarrow 6 \text{ eqns in 5 variables} \]

B, C, D, E, F

NB: no exact solution!

Q: Best approximate?

In a population of rabbits:

(1) Half survive their first year
(2) Half of those survive the 2nd year
(3) Max life span is 3 years
(4) Produce 0, 6, 8 offspring in years 1, 2, 3, resp.
Q: How many rabbits will there be after 100 years?

\[ x_{2020} = \# \text{1st year rabbits in 2020} \]
\[ y_{2020} = \# \text{2nd year rabbits in 2020} \]
\[ z_{2020} = \# \text{3rd year rabbits in 2020} \]

\[ \begin{align*}
  y_{2021} &= \frac{1}{2} x_{2020} \\
  z_{2021} &= \frac{1}{2} y_{2020} \\
  w_{2021} &= 6y_{2020} + 8z_{2020}
\end{align*} \]

Geometry of Solutions

• One eqn in 2 vars:
  \[ x - 2y = 1 \Rightarrow y = \frac{1}{2}x - \frac{1}{2} \]

• One eqn in 3 vars:
  \[ x + y + z = 1 \Rightarrow z = 1 - x - y \]

• One eqn in 4 vars:
  \[ x + y + z + w = 1 \Rightarrow \text{"3-plane in \(xyz\)-space"} \]

More equations?

• 2 eqns in 2 vars
  \[ \begin{align*}
    x - 2y &= 1 \\
    3x + 2y &= 11
  \end{align*} \]
* 2 eqns in 3 vars:
  \[ x + y + z = 1 \]
  \[ x - z = 0 \]
  intersection of 2 planes in xyz-space: line

* 3 eqns in 3 vars:
  \[ x + y + z = 1 \]
  \[ x - z = 0 \]
  \[ y = 0 \]
  \[ x = \frac{1}{2} \]
  \[ y = 0 \]
  \[ z = \frac{1}{2} \]
  one point = intersection of 3 planes

Other possibilities?

Administrative
- Course site & Sakai
- Please turn cameras on!!
- Piazza: tab in Sakai
- Office hours!
- Scan your homework!  
  → see Syllabus
- Textbook:  
  - Strang
  - IIA
- Recorded lectures  
  → Joe & Jesse.
- Groups & quizzes
- Exams