Exercises from Strang:

Problem Set 4.1 #24, 28, 29
Problem Set 4.2 #2, 5–7, 11, 12, 17\(^1\), 19, 20, 24, 26, 31
Problem Set 4.3 #28

Additional Problem:

1. You may use Sage for any or all parts of this question. Be sure to show all your work. For example, if you used Sage to compute the RREF of a matrix, show the RREF and explain what conclusions you draw from it that answer the relevant question.

https://sagecell.sagemath.org/

Consider the equation \( A\vec{x} = \vec{c} \) with

\[
A = \begin{pmatrix} 1 & -1 & 5 & -4 \\ 2 & 0 & 4 & -2 \\ 3 & 2 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \vec{c} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}.
\]

a) Show that \( \vec{c} \notin C(A) \).

b) Compute \( A^T A \) and \( A^T \vec{c} \).

c) Why does the normal equation \( A^T A\vec{x} = A^T \vec{c} \) not have a unique solution?

d) Find the general solution of the normal equation.

e) Find the projection \( \vec{p} \) of \( \vec{c} \) onto \( C(A) \) and the error vector \( \vec{e} \in N(A^T) \).

f) Find the projection matrix \( P \) onto \( C(A) \), and check that \( P\vec{c} = \vec{p} \). (Hint: see question 4.3.28 in the book.)