MATH 1553-C
MIDTERM EXAMINATION 3

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Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA scans your exam.
- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work unless instructed otherwise.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

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Problem 1.

[2 points each]

In the following questions, if the statement is true, circle **T**; if it is always false, circle **F**.

All matrices are assumed to have real entries.

a) **T** **F** Every 2×2 matrix with eigenvalues 0 and 1 is diagonalizable.

b) **T** **F** Every 3×3 matrix with eigenvalues 0 and 1 is invertible.

c) **T** **F** If A is a square matrix and $\det(A) \neq 0$, then the rows of A are linearly independent.

d) **T** **F** Every upper-triangular square matrix is diagonalizable.

e) **T** **F** Every upper-triangular square matrix does not have any complex eigenvalues.

[Scratch work for problem 1]

Problem 2. [2 points each]

In this problem, it is not necessary to show your work or justify your answers.

a) Give an example of a 2×2 matrix which is neither invertible nor diagonalizable.

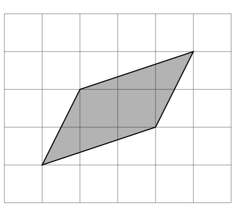
b) Which of the following 3×3 matrices with real entries are necessarily diagonalizable? Circle all that apply.

1. A matrix with three distinct real eigenvalues.
2. A matrix with three distinct real eigenvectors.
3. A matrix with eigenvalues 2 and 3 such that $\lambda = 3$ has geometric multiplicity equal to 2.
4. A matrix with three linearly independent eigenvectors.

c) Write a correct definition of an eigenvector:

"v is an eigenvector of an $n \times n$ matrix A provided that

d) What is the area of the parallelogram in the picture? (The grid marks are spaced one unit apart.)



e) Let A be a 4×4 matrix with determinant 3. What is $\det(-A)$?

[Scratch work for problem 2]

Problem 3.

Consider the matrix

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

a) [3 points] Find the characteristic polynomial $f(\lambda)$ of A.

b) [3 points] Find all eigenvalues of A, real and complex.

c) [4 points] For each eigenvalue, find a corresponding (real or complex) eigenvector.

[Scratch work for problem 3]

Problem 4.

The matrix

$$A = \begin{pmatrix} 3 & -4 & -7 \\ -2 & 5 & 7 \\ 2 & -4 & -6 \end{pmatrix}$$

has characteristic polynomial $f(\lambda) = -\lambda(\lambda - 1)^2$

a) [4 points] Find bases for the 0-eigenspace and the 1-eigenspace.

b) [2 points] What are the algebraic and geometric multiplicities of the eigenvalues?

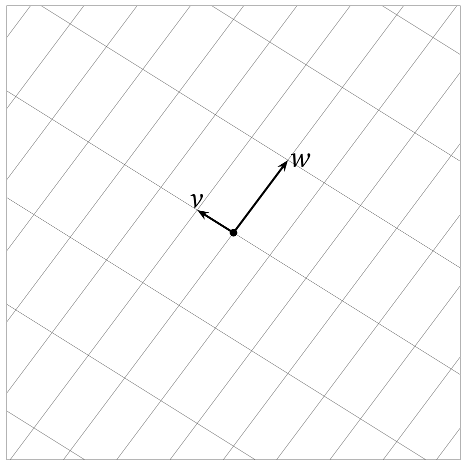
c) [3 points] Find a diagonal matrix D and an invertible matrix C such that $A = CDC^{-1}$, or explain why A is not diagonalizable.

d) [1 point] Write a nonzero vector in Nul$A$.

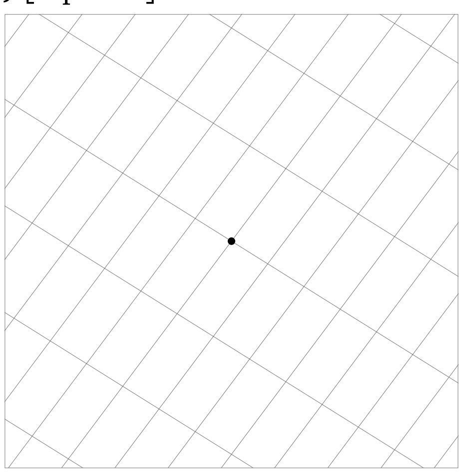
[Scratch work for problem 4]

Problem 5.

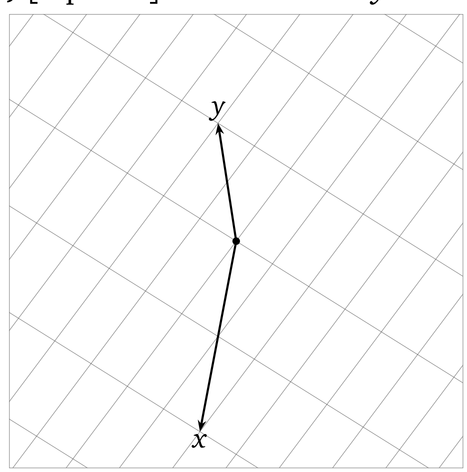
A certain 2×2 matrix A has eigenectors v and w, pictured below, with corresponding eigenvalues $3/2$ and $-1/2$.



a) [3 points] Draw Av and Aw below.



b) [4 points] Draw Ax and Ay below.



c) [3 points] Compute $A^{100}(3v + 4w)$. (Your answer will be in terms of v and w.)

[Scratch work for problem 5]

[Additional scratch work]

[Additional scratch work]