1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that $A$ is an $n \times n$ matrix.

a) The entries on the main diagonal of $A$ are the eigenvalues of $A$.

b) To find the eigenvectors of $A$, we reduce the matrix $A$ to row echelon form.

c) If $v_1$ and $v_2$ are linearly independent eigenvectors of $A$, then they must correspond to different eigenvalues.

d) If $A$ is invertible and 2 is an eigenvalue of $A$, then $\frac{1}{2}$ is an eigenvalue of $A^{-1}$.

2. In what follows, $T$ is a linear transformation with matrix $A$. Find the eigenvectors and eigenvalues of $A$ without doing any matrix calculations. (Draw a picture!)

a) $T =$ projection onto the $xz$-plane in $\mathbb{R}^3$. 

b) $T = \text{reflection over } y = 2x \text{ in } \mathbb{R}^2.$

3. Consider the matrix

$$A = \frac{1}{5} \begin{pmatrix} 8 & 3 \\ 2 & 7 \end{pmatrix}.$$ 

Find, draw, and label the eigenspaces of $A.$