Math 1553 Worksheet §3.3 and §3.4, Matrix Equations and Solution Sets

For problems 1 and 2 below: The professor in your widgets and gizmos class is trying to decide between three different grading schemes for computing your final course grade. The schemes are based on homework (HW), quiz grades (Q), midterms (M), and a final exam (F). The three schemes can be described by the following matrix $A$:

$$
\begin{pmatrix}
\text{HW} & \text{Q} & \text{M} & \text{F} \\
\text{Scheme 1} & 0.1 & 0.1 & 0.5 & 0.3 \\
\text{Scheme 2} & 0.1 & 0.1 & 0.4 & 0.4 \\
\text{Scheme 3} & 0.1 & 0.1 & 0.6 & 0.2 \\
\end{pmatrix}
$$

Feel free to use a calculator to carry out arithmetic in problems 1 and 2.

1. Suppose that you have a score of $x_1$ on homework, $x_2$ on quizzes, $x_3$ on midterms, and $x_4$ on the final, with potential final course grades of $b_1$, $b_2$, $b_3$. Write a matrix equation $Ax = b$ to relate your final grades to your scores.

2. Suppose that you end up with averages of 90% on the homework, 90% on quizzes, 85% on midterms, and a 95% score on the final exam. Use Problem 1 to determine which grading scheme leaves you with the highest overall course grade.
3. If the statement is always true, circle True. Otherwise, circle False. Justify your answer.
   a) If $A$ is a $5 \times 4$ matrix, then the equation $Ax = b$ must be inconsistent for some $b$ in $\mathbb{R}^5$.
      TRUE      FALSE

   b) Any linear combination of vectors can always be written in the form $Ax$ for a suitable matrix $A$ and vector $x$.
      TRUE      FALSE

4. Find the solution sets of $x_1 - 3x_2 + 5x_3 = 0$ and $x_1 - 3x_2 + 5x_3 = 3$ and write them in parametric vector form. How do the solution sets compare geometrically?