MATH 1553
QUIZ #2: §§2.2, 2.3

Name		Section	

1. [5 points] Put the following matrix into reduced row echelon form using elementary row operations. Show your work.

$$\begin{pmatrix} -3 & 2 & 0 & -1 \\ 3 & -4 & 5 & -4 \\ 1 & -1 & 1 & -1 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

Solution.

$$\begin{pmatrix} -3 & 2 & 0 & -1 \\ 3 & -4 & 5 & -4 \\ 1 & -1 & 1 & -1 \\ 4 & -2 & -2 & 4 \end{pmatrix} \xrightarrow{R_1 \longleftrightarrow R_3} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & -4 & 5 & -4 \\ -3 & 2 & 0 & -1 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

$$\xrightarrow{R_2 = R_2 - 3R_1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ -3 & 2 & 0 & -1 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

$$\xrightarrow{R_3 = R_3 + 3R_1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 3 & -4 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

$$\xrightarrow{R_4 = R_4 - 4R_1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 3 & -4 \\ 0 & 2 & -6 & 8 \end{pmatrix}$$

$$\xrightarrow{R_2 = R_2 \times -1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 3 & -4 \\ 0 & 2 & -6 & 8 \end{pmatrix}$$

$$\xrightarrow{R_3 = R_3 + R_2} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 2 & -6 & 8 \end{pmatrix}$$

$$\xrightarrow{R_4 = R_4 - 2R_2} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -2 & 6 \end{pmatrix}$$

$$\xrightarrow{R_4 = R_4 + 2R_3} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_2 = R_2 + 2R_3} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 = R_1 - R_3} \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 = R_1 + R_2} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2. [1 point each] For each of the following augmented matrices, circling $0, 1$, or ∞ to indicate how many solutions the corresponding system of linear equations has.

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 5 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 5 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

0 1 ∞ 0 1 ∞ 0 1 ∞

$$\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 5 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

0 1 ∞ 0 1 ∞

Solution.

The first, third, and fifth matrices each have a free variable in columns 2, 1, and 2, respectively. None of these has a pivot in the last (augmented) column, so they correspond to systems with infinitely many solutions. The second matrix has a pivot in the last column, so it corresponds to an inconsistent system. The fourth matrix has a unique solution.