Everything you’ll need to know is on the master website:
http://people.math.gatech.edu/~cjankowski3/18f/m1553/webpage/
or on the website for this section:
http://people.math.gatech.edu/~jrabinoff/1819F-1553/
(There are links on Canvas.) **Read them! Bookmark them!** Chances are, all your (non-math) questions are answered there.

Warmup assignment is due on Friday at 11:59pm on WeBWorK.

Enroll in Piazza (the link is on Canvas). You can ask questions there, and we will use it for in-class polling on a daily basis. **Please use your Canvas email address to enroll**, so that your poll responses show up in the Canvas gradebook.

It’s probably easiest to respond to polls using a smartphone. Download the Piazza app.

My office is Skiles 244 and Rabinoffice hours are: Mondays, 12–1pm; Wednesdays, 1–3pm.

Your TAs have office hours too. You can go to any of them. Details on the website.
Chapter 2

Systems of Linear Equations: Algebra
Section 2.1

Systems of Linear Equations
Recall that $\mathbb{R}$ denotes the collection of all real numbers, i.e. the number line. It contains numbers like 0, $-1$, $\pi$, $\frac{3}{2}$, ... 

**Definition**

Let $n$ be a positive whole number. We define

$$\mathbb{R}^n = \text{all ordered } n\text{-tuples of real numbers } (x_1, x_2, x_3, \ldots, x_n).$$

**Example**

When $n = 1$, we just get $\mathbb{R}$ back: $\mathbb{R}^1 = \mathbb{R}$. Geometrically, this is the *number line*.

![Number line diagram](image-url)
Example

When \( n = 2 \), we can think of \( \mathbb{R}^2 \) as the *plane*. This is because every point on the plane can be represented by an ordered pair of real numbers, namely, its \( x \)- and \( y \)-coordinates.
Example
When \( n = 3 \), we can think of \( \mathbb{R}^3 \) as the space we (appear to) live in. This is because every point in space can be represented by an ordered triple of real numbers, namely, its \( x \)-, \( y \)-, and \( z \)-coordinates.
So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

... go back to the definition: ordered $n$-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They’re still “geometric” spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they’re harder to visualize.

We’ll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we’ll only draw pictures for $\mathbb{R}^2$ and $\mathbb{R}^3$.

The power of using these spaces is the ability to use elements of $\mathbb{R}^n$ to label various objects of interest, like solutions to systems of equations.
All colors you can see can be described by three quantities: the amount of red, green, and blue light in that color. Therefore, we can use the elements of $\mathbb{R}^3$ to label all colors: the point $(.2, .4, .9)$ labels the color with 20% red, 40% green, and 90% blue.
Labeling with $\mathbb{R}^n$

Example

Last time we could have used $\mathbb{R}^4$ to *label* the amount of traffic $(x, y, z, w)$ passing through four streets.

For instance the point $(100, 20, 30, 150)$ corresponds to a situation where 100 cars per hour drive on road $x$, 20 cars per hour drive on road $y$, etc.
What does the solution set of a linear equation look like?

$x + y = 1 \implies \text{a line in the plane: } y = 1 - x$
This is called the **implicit equation** of the line.

We can write the same line in **parametric form** in $\mathbb{R}^2$:

$$(x, y) = (t, 1 - t) \quad t \text{ in } \mathbb{R}.$$  

This means that every point on the line has the form $(t, 1 - t)$ for some real number $t$. Note we are using $\mathbb{R}$ to *label* the points on a line in $\mathbb{R}^2$.

**Aside**
What is a line? A ray that is *straight* and infinite in both directions.
What does the solution set of a linear equation look like?

\[ x + y + z = 1 \] 

\( \implies \) a plane in space:
This is the **implicit equation** of the plane.

Does this plane have a **parametric form**?

\[ (x, y, z) = (t, w, 1 - t - w) \quad t, w \text{ in } \mathbb{R}. \]

Note we are using \( \mathbb{R}^2 \) to *label* the points on a plane in \( \mathbb{R}^3 \).

**Aside**
What is a plane? A flat sheet of paper that’s infinite in all directions.
What does the solution set of a linear equation look like?

\[ x + y + z + w = 1 \implies \text{a “3-plane” in “4-space”…} \] [not pictured here]
Everybody get out your gadgets!

No! Every point on this plane is in $\mathbb{R}^3$: that means it has three coordinates. For instance, $(1, 0, 0)$. Every point in $\mathbb{R}^2$ has two coordinates. But we can label the points on the plane by $\mathbb{R}^2$.  

Poll

Is the plane from the previous example equal to $\mathbb{R}^2$?

A. Yes  
B. No
Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

\[ x - 3y = -3 \]
\[ 2x + y = 8 \]

... is the *intersection* of two lines, which is a *point* in this case.

In general it’s an intersection of lines, planes, etc.

[two planes intersecting]
Kinds of Solution Sets

In what other ways can two lines intersect?

\[ x - 3y = -3 \]
\[ x - 3y = 3 \]

has no solution: the lines are parallel.

A system of equations with no solutions is called inconsistent.
Kinds of Solution Sets

In what other ways can two lines intersect?

\[
x - 3y = -3 \\
2x - 6y = -6
\]

has infinitely many solutions: they are the *same line*.

Note that multiplying an equation by a nonzero number gives the *same solution set*. In other words, they are *equivalent* (systems of) equations.
What about in three variables?

In how many different ways can three planes intersect in space?

A. One
B. Two
C. Three
D. Four
E. Five
F. Six
G. Seven
H. Eight
Summary

- $\mathbb{R}^n$ is the set of ordered lists of $n$ numbers.
- $\mathbb{R}^n$ can be used to label geometric objects, like $\mathbb{R}^2$ can label points on a plane.
- The solutions of a system equations look like an intersection of lines, planes, etc.
- Finding all the solutions of a system of equations means finding a **parametric form**: a labeling by some $\mathbb{R}^n$. 