Everything you’ll need to know is on the master website:
http://people.math.gatech.edu/~cjankowski3/18f/m1553/webpage/
or on the website for this section:
http://people.math.gatech.edu/~jrabinoff/1819F-1553/
(There are links on Canvas.) **Read them! Bookmark them!** Chances are, all your (non-math) questions are answered there.

Warmup assignment is due on Friday at 11:59pm on WeBWorK.

Enroll in Piazza (the link is on Canvas). You can ask questions there, and we will use it for in-class polling on a daily basis. **Please use your Canvas email address to enroll**, so that your poll responses show up in the Canvas gradebook.

It’s probably easiest to respond to polls using a smartphone. Download the Piazza app.

My office is Skiles 244 and Rabinoffice hours are: Mondays, 12–1pm; Wednesdays, 1–3pm.

Your TAs have office hours too. You can go to any of them. Details on the website.
Chapter 2

Systems of Linear Equations: Algebra
Section 2.1

Systems of Linear Equations
Recall that $\mathbb{R}$ denotes the collection of all real numbers, i.e. the number line. It contains numbers like $0, -1, \pi, \frac{3}{2}, \ldots$

**Definition**

Let $n$ be a positive whole number. We define

$$\mathbb{R}^n = \text{all ordered } n\text{-tuples of real numbers } (x_1, x_2, x_3, \ldots, x_n).$$

**Example**

When $n = 1$, we just get $\mathbb{R}$ back: $\mathbb{R}^1 = \mathbb{R}$. Geometrically, this is the *number line*.
Example
When $n = 2$, we can think of $\mathbb{R}^2$ as the plane. This is because every point on the plane can be represented by an ordered pair of real numbers, namely, its $x$- and $y$-coordinates.
Example

When \( n = 3 \), we can think of \( \mathbb{R}^3 \) as the *space* we (appear to) live in. This is because every point in space can be represented by an ordered triple of real numbers, namely, its \( x \)-, \( y \)-, and \( z \)-coordinates.

\[
(1, -1, 3) \quad (-2, 2, 2)
\]
So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

...go back to the definition: ordered $n$-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They’re still “geometric” spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they’re harder to visualize.

We’ll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we’ll only draw pictures for $\mathbb{R}^2$ and $\mathbb{R}^3$.

The power of using these spaces is the ability to use elements of $\mathbb{R}^n$ to label various objects of interest, like solutions to systems of equations.
All colors you can see can be described by three quantities: the amount of red, green, and blue light in that color. Therefore, we can use the elements of $\mathbb{R}^3$ to label all colors: the point $(0.2, 0.4, 0.9)$ labels the color with 20% red, 40% green, and 90% blue.
Last time we could have used $\mathbb{R}^4$ to label the amount of traffic $(x, y, z, w)$ passing through four streets.

For instance the point $(100, 20, 30, 150)$ corresponds to a situation where 100 cars per hour drive on road $x$, 20 cars per hour drive on road $y$, etc.
What does the solution set of a linear equation look like?

\[ x + y = 1 \] → a line in the plane: \[ y = 1 - x \]

This is called the \textbf{implicit equation} of the line.

We can write the same line in \textbf{parametric form} in \( \mathbb{R}^2 \):

\[ (x, y) = (t, 1 - t) \quad t \text{ in } \mathbb{R}. \]

This means that every point on the line has the form \((t, 1 - t)\) for some real number \(t\). Note we are using \( \mathbb{R} \) to \textit{label} the points on a line in \( \mathbb{R}^2 \).

\textbf{Aside}

What is a line? A ray that is \textit{straight} and infinite in both directions.
What does the solution set of a linear equation look like?

\[ x + y + z = 1 \rightarrow \text{a plane in space:} \]

This is the **implicit equation** of the plane.

Does this plane have a **parametric form**?

\[(x, y, z) = (t, w, 1 - t - w) \quad t, w \text{ in } \mathbb{R}.\]

Note we are using \(\mathbb{R}^2\) to *label* the points on a plane in \(\mathbb{R}^3\).

**Aside**

What is a plane? A flat sheet of paper that’s infinite in all directions.
What does the solution set of a linear equation look like?

\[ x + y + z + w = 1 \rightarrow \text{a “3-plane” in “4-space”…} \]
Everybody get out your gadgets!
Systems of Linear Equations

What does the solution set of a system of more than one linear equation look like?

\[ x - 3y = -3 \]
\[ 2x + y = 8 \]

In general it’s an intersection of lines, planes, etc.

[two planes intersecting]
In what other ways can two lines intersect?

\[ x - 3y = -3 \]
\[ x - 3y = 3 \]

A system of equations with no solutions is called **inconsistent**.
Kinds of Solution Sets

In what other ways can two lines intersect?

\[ x - 3y = -3 \]
\[ 2x - 6y = -6 \]

Note that multiplying an equation by a nonzero number gives the same solution set. In other words, they are equivalent (systems of) equations.
What about in three variables?
Summary

- $\mathbb{R}^n$ is the set of ordered lists of $n$ numbers.
- $\mathbb{R}^n$ can be used to label geometric objects, like $\mathbb{R}^2$ can label points on a plane.
- The solutions of a system of equations look like an intersection of lines, planes, etc.
- Finding all the solutions of a system of equations means finding a **parametric form**: a labeling by some $\mathbb{R}^n$. 