Math 1553 Worksheet §§5.3, 5.5

1. Answer yes / no / maybe. In each case, A is a matrix with real entries.
   a) If $A$ is a $3 \times 3$ matrix with characteristic polynomial $-\lambda(\lambda - 5)^2$, then the 5-eigenspace is 2-dimensional.
   b) If $A$ is an invertible $2 \times 2$ matrix, then $A$ is diagonalizable.
   c) Can a $3 \times 3$ matrix $A$ have a non-real complex eigenvalue with multiplicity 2?
   d) Can a $3 \times 3$ matrix $A$ have eigenvalues 3, 5, and $2 + i$?

2. Let $A = \begin{pmatrix} 8 & 36 & 62 \\ -6 & -34 & -62 \\ 3 & 18 & 33 \end{pmatrix}$.

   The characteristic polynomial for $A$ is $f(\lambda) = -\lambda^3 + 7\lambda^2 - 16\lambda + 12$. Decide if $A$ is diagonalizable. If it is, find an invertible matrix $P$ and a diagonal matrix $D$ such that $A = PDP^{-1}$.

3. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.
   a) Find all (real and) eigenvalues and eigenvectors of $A$. 
b) (After finishing §5.5 in lecture.) Write $A = P C P^{-1}$, where $C$ is a rotation followed by a scale. Describe what $A$ does geometrically. Draw a picture.

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**Supplemental Problems**

These are additional practice problems after completing the worksheet.

1. Let $A$ and $B$ be $3 \times 3$ real matrices. Answer yes / no / maybe:
   a) If $A$ and $B$ have the same eigenvalues, then $A$ is similar to $B$.
   b) If $A$ and $B$ both have eigenvalues $-1, 0, 1$, then $A$ is similar to $B$.
   c) If $A$ is diagonalizable and invertible, then $A^{-1}$ is diagonalizable.

2. Give an example of a non-diagonal $2 \times 2$ matrix which is diagonalizable but not invertible. Justify your answer.

3. Suppose $A$ is a $7 \times 7$ matrix with four distinct eigenvalues. One eigenspace has dimension 2, while another eigenspace has dimension 3. Is it possible that $A$ is not diagonalizable?

4. Let $A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$.
   a) Find all (complex) eigenvalues and eigenvectors of $A$.
   b) Write $A = P C P^{-1}$, where $C$ is a block diagonal matrix, as in the slides near the end of section 5.5.
   c) What does $A$ do geometrically? Draw a picture.