1. [5 points] Write a mathematically correct definition of an eigenvalue. Pay attention to your quantifiers.

   “λ is an eigenvalue of an \( n \times n \) matrix \( A \) provided that there exists a nonzero solution \( v \) to the equation \( Av = \lambda v \).”

2. [4 points] Consider the matrix \( A \) for the transformation that reflects over a line \( L \). Find all eigenvalues of \( A \), and draw a picture of an eigenvector for each eigenvalue in the box below.

   Solution.

   The only vectors that are taken to a scalar multiple are the vectors on \( L \), which are not moved, and the vectors perpendicular to \( L \), which are negated. The former have eigenvalue 1, and the latter have eigenvalue \(-1\).

3. [3 points] Find all eigenvalues of \( A \).

   \[
   A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}
   \]

   Solution.

   The characteristic polynomial of \( A \) is

   \[
   f(\lambda) = \lambda^2 - \text{Tr}(A)\lambda + \det(A) = \lambda^2 - 3\lambda - 1.
   \]

   The roots are

   \[
   \lambda = \frac{3 \pm \sqrt{9 + 4}}{2} = \frac{3 \pm \sqrt{13}}{2}.
   \]

   These are the eigenvalues of \( A \).