Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.
In this problem, $A$ is an $m \times n$ matrix ($m$ rows and $n$ columns) and $b$ is a vector in $\mathbb{R}^m$. Circle $T$ if the statement is always true (for any choices of $A$ and $b$) and circle $F$ otherwise. Do not assume anything else about $A$ or $b$ except what is stated.

a) T F The matrix below is in reduced row echelon form.
\[
\begin{pmatrix}
1 & 1 & 0 & -3 & 1 \\
0 & 0 & 1 & -1 & 5 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

b) T F If $A$ has fewer than $n$ pivots, then $Ax = b$ has infinitely many solutions.

c) T F If the columns of $A$ span $\mathbb{R}^m$, then $Ax = b$ must be consistent.

d) T F If $Ax = b$ is consistent, then the equation $Ax = 5b$ is consistent.

e) T F If $Ax = b$ is consistent, then the solution set is a span.
Acme Widgets, Gizmos, and Doodads has two factories. Factory A makes 10 widgets, 3 gizmos, and 2 doodads every hour, and factory B makes 4 widgets, 1 gizmo, and 1 doodad every hour.

a) If factory A runs for $a$ hours and factory B runs for $b$ hours, how many widgets, gizmos, and doodads are produced? Express your answer as a vector equation.

b) A customer places an order for 16 widgets, 5 gizmos, and 3 doodads. Can Acme fill the order with no widgets, gizmos, or doodads left over? If so, how many hours do the factories run? If not, why not?
Consider the system below, where $h$ and $k$ are real numbers.

\[
\begin{align*}
x + 3y &= 2 \\
3x - hy &= k.
\end{align*}
\]

a) Find the values of $h$ and $k$ which make the system inconsistent.

b) Find the values of $h$ and $k$ which give the system a unique solution.

c) Find the values of $h$ and $k$ which give the system infinitely many solutions.
Consider the following consistent system of linear equations.

\[
\begin{align*}
x_1 + 2x_2 + 3x_3 + 4x_4 &= -2 \\
3x_1 + 4x_2 + 5x_3 + 6x_4 &= -2 \\
5x_1 + 6x_2 + 7x_3 + 8x_4 &= -2
\end{align*}
\]

a) [4 points] Find the parametric vector form for the general solution.

b) [3 points] Find the parametric vector form of the corresponding homogeneous equations. [Hint: you’ve already done the work.]

c) [3 points] Unrelated to parts (a) and (b).
If \(b, v, w\) are vectors in \(\mathbb{R}^3\) and \(\text{Span}\{b, v, w\} = \mathbb{R}^3\), is it possible that \(b\) is in \(\text{Span}\{v, w\}\)? Fully justify your answer.
Problem 5. [10 points]

The diagram below describes traffic in a part of town.

Traffic flow (cars/hr)

110  x_3  40
  
  x_3  x_1

200  x_2  320

90  40

a) Write a system of three linear equations in \( x_1, x_2, \) and \( x_3 \) corresponding to the traffic flow.

b) Use an augmented matrix to solve this system of linear equations. Were we given enough information to know the exact values of \( x_1, x_2, \) and \( x_3 \)?
[Scratch work]