Please read all instructions carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!
Scoring Page

Please do not write on this page.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
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**Problem 1.**

In parts (c) and (e), $A$ denotes an $m \times n$ matrix ($m$ rows and $n$ columns), and in part (c), $b$ is a vector in $\mathbb{R}^m$. In (b)–(e), circle **T** if the statement is necessarily true, and circle **F** otherwise.

a) What is the best way to describe the solution set of the equation $x + 2y = 0$?

<table>
<thead>
<tr>
<th>a line in $\mathbb{R}^2$</th>
<th>a line in $\mathbb{R}^3$</th>
<th>a plane in $\mathbb{R}^2$</th>
<th>a plane in $\mathbb{R}^3$</th>
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<tbody>
<tr>
<td><strong>T</strong></td>
<td>F</td>
<td>F</td>
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</table>

b) **T**   **F**  

The following matrix is in row echelon form:

\[
\begin{pmatrix}
1 & 7 & 2 & 4 \\
0 & 0 & 1 & -2 \\
0 & 0 & 0 & 15
\end{pmatrix}
\]

c) **T**   **F**  

If $A$ has a pivot in every column, then the matrix equation $Ax = b$ is consistent.

d) **T**   **F**  

The following matrix corresponds to a linear system with two free variables:

\[
\begin{pmatrix}
1 & 7 & 2 & 4 \\
0 & 0 & 1 & -2 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

e) **T**   **F**  

The solution set of $Ax = 0$ is a span in $\mathbb{R}^m$. 
Problem 2.

Consider the following system of linear equations:

\[\begin{align*}
3x + 7y + 4z &= -4 \\
x + 2y + 2z &= -1.
\end{align*}\]

(a) [1 point] Write the system as a vector equation.

(b) [1 point] Write the system as a matrix equation.

(c) [1 point] Write the system as an augmented matrix.

(d) [4 points] Find the solution set in parametric vector form.

(e) [3 points] Draw a picture of the solution set.
Problem 3.

Consider the following vectors:

\[ v_1 = \begin{pmatrix} 2\pi \\ -7 \\ 114 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 13 \\ 11/2 \end{pmatrix}. \]

a) [4 points] Describe \( \text{Span}\{v_1, v_2\} \) geometrically: “it is a \[ \square \] in \( \mathbb{R}^\square \).”

b) [6 points] Find a matrix \( A \) with three rows, with the property that the matrix equation \( Ax = b \) is consistent if and only if \( b \) is in \( \text{Span}\{v_1, v_2\} \).
Problem 4. [5 points each]

a) Is \( \begin{pmatrix} 4 \\ 15 \\ -8 \\ -1 \end{pmatrix} \) in \( \text{Span} \left\{ \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ 0 \\ 1 \end{pmatrix} \right\} \)?

b) Find a vector in \( \mathbb{R}^3 \) that is not in \( \text{Span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \).
Problem 5.

Consider the following picture of two vectors $v, w$:

a) For each of the labeled points, estimate the coefficients $x, y$ such that the linear combination $xv + yw$ is the vector ending at that point.

\[
\begin{align*}
\quad v + \quad w &= a \\
\quad v + \quad w &= b \\
\quad v + \quad w &= c \\
\quad v + \quad w &= d \\
\quad v + \quad w &= e
\end{align*}
\]

b) Find two vectors $p, q$ in $\mathbb{R}^2$ such that none of the points $a, b, c, d, e$ is in $\text{Span}\{p, q\}$.

You needn’t show your work in this problem.
[Scratch work]