Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!
Scoring Page

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Problem 1. [2 points each]

In parts (c) and (e), \( A \) denotes an \( m \times n \) matrix (\( m \) rows and \( n \) columns), and in (e), \( b \) is a vector in \( \mathbb{R}^m \). In (b)–(e), circle \( T \) if the statement is necessarily true, and circle \( F \) otherwise.

a) What is the best way to describe the solution set of the equation \( x + 2y - z = 0 \)?
   
   a line in \( \mathbb{R}^2 \)  
   a line in \( \mathbb{R}^3 \)  
   a plane in \( \mathbb{R}^2 \)  
   a plane in \( \mathbb{R}^3 \)

b)  \( T \)  \( F \)  The following matrix has three pivots:
   
   \[
   \begin{pmatrix}
   1 & 7 & 2 & 4 \\
   0 & 0 & 1 & -2 \\
   0 & 0 & 0 & 15
   \end{pmatrix}
   \]

c)  \( T \)  \( F \)  It is possible for the matrix equation \( Ax = 0 \) to be inconsistent.

d)  \( T \)  \( F \)  The following matrix corresponds to a linear system with one free variable:
   
   \[
   \begin{pmatrix}
   1 & 7 & 2 & 4 \\
   0 & 0 & 1 & -2 \\
   0 & 0 & 0 & 0
   \end{pmatrix}
   \]

e)  \( T \)  \( F \)  The solution set of \( Ax = b \) is empty or it is a translate of a span in \( \mathbb{R}^m \).
Problem 2.

Consider the following system of linear equations:
\[\begin{align*}
2x + y + 12z &= 1 \\
x + 2y + 9z &= -1.
\end{align*}\]

a) [1 point] Write the system as a vector equation.

b) [1 point] Write the system as a matrix equation.

c) [1 point] Write the system as an augmented matrix.

d) [4 points] Find the solution set in parametric vector form.

e) [3 points] Draw a picture of the solution set.
Problem 3.

Consider the following vectors:

\[ v_1 = \begin{pmatrix} 17 \\ -3 \\ 24 \end{pmatrix} \quad v_2 = \begin{pmatrix} 7/2 \\ 0 \\ \pi \end{pmatrix}. \]

a) [4 points] Describe \( \text{Span}\{v_1, v_2\} \) geometrically: “it is a \( \square \) in \( \mathbb{R}^3 \).”

b) [6 points] Find a matrix \( A \) with three rows, with the property that the matrix equation \( Ax = b \) is consistent if and only if \( b \) is in \( \text{Span}\{v_1, v_2\} \).
Problem 4.  

[5 points each]

a) Is \[
\begin{pmatrix}
-1 \\
-5 \\
12 \\
4
\end{pmatrix}
\] in \( \text{Span} \{ \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 7 \\ 0 \\ 1 \end{pmatrix} \} \)?

b) Find a vector in \( \mathbb{R}^3 \) that is not in \( \text{Span} \{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \} \).
Problem 5.

Consider the following picture of two vectors $v, w$:

```
Consider the following picture of two vectors $v, w$:

\[ v \]
\[ w \]
\[ a \]
\[ b \]
\[ c \]
\[ d \]
\[ e \]
```

a) For each of the labeled points, estimate the coefficients $x, y$ such that the linear combination $xv + yw$ is the vector ending at that point.

\[
\begin{align*}
_____ v + _____ w &= a \\
_____ v + _____ w &= b \\
_____ v + _____ w &= c \\
_____ v + _____ w &= d \\
_____ v + _____ w &= e
\end{align*}
\]

b) Find two vectors $p, q$ in $\mathbb{R}^2$ such that none of the points $a, b, c, d, e$ is in $\text{Span}\{p, q\}$.

You needn't show your work in this problem.
[Scratch work]