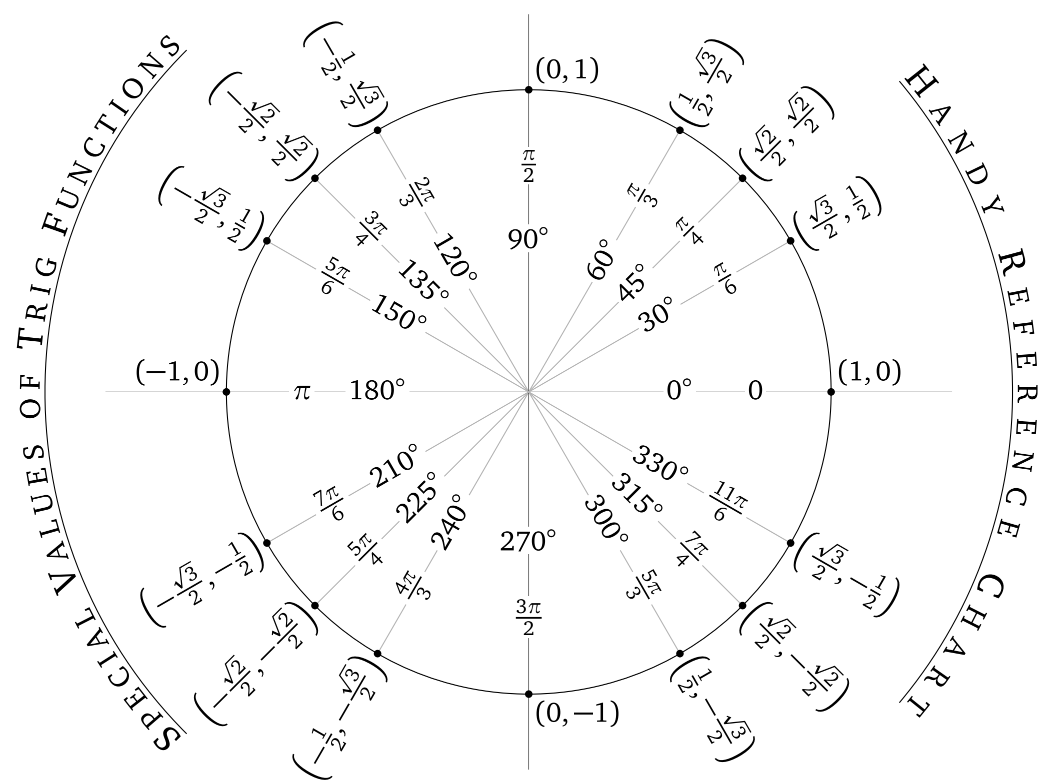
MATH 1553-B
MIDTERM EXAMINATION 3

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work unless instructed otherwise.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!



Problem 1. [2 points each]

In this problem, if the statement is always true, circle **T**; otherwise, circle **F**.

a) **T** **F** If A is row equivalent to B, then A and B have the same eigenvalues.

b) **T** **F** If A is similar to B, then A and B have the same characteristic polynomial.

c) **T** **F** If A is similar to B, then A and B have the same eigenvectors.

d) **T** **F** If A is diagonalizable, then A has n distinct eigenvalues.

e) **T** **F** Every square matrix is diagonalizable if we allow complex eigenvalues and eigenvectors.

Problem 2.

In this problem, you need not explain your answers; just circle the correct one(s).

Let A be an $n \times n$ matrix.

a) [3 points] Which **one** of the following statements is correct?

1. An eigenvector of A is a vector v such that $Av = \lambda v$ for a nonzero scalar λ.
2. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a scalar λ.
3. An eigenvector of A is a nonzero scalar λ such that $Av = \lambda v$ for some vector v.
4. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a nonzero scalar λ.

b) [3 points] Which **one** of the following statements is **not** correct?

1. An eigenvalue of A is a scalar λ such that $A - \lambda I$ is not invertible.
2. An eigenvalue of A is a scalar λ such that $(A - \lambda I)v = 0$ has a solution.
3. An eigenvalue of A is a scalar λ such that $Av = \lambda v$ for a nonzero vector v.
4. An eigenvalue of A is a scalar λ such that $\det(A - \lambda I) = 0$.

c) [4 points] Which of the following 3×3 matrices are necessarily diagonalizable over the real numbers? (Circle all that apply.)

1. A matrix with three distinct real eigenvalues.
2. A matrix with one real eigenvalue.
3. A matrix with a real eigenvalue λ of algebraic multiplicity 2, such that the λ-eigenspace has dimension 2.
4. A matrix with a real eigenvalue λ such that the λ-eigenspace has dimension 2.

Problem 3.

Consider the matrix

$$A = \begin{pmatrix} -1 & -4 & 0 \\ 1 & 3 & 0 \\ 7 & 10 & 2 \end{pmatrix}.$$

a) [4 points] Find the eigenvalues of A, and compute their algebraic multiplicities.

b) [4 points] For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) [2 points] Is A diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. If not, why not?

Problem 4.

Consider the matrix

$$A = \begin{pmatrix} 3\sqrt{3}-1 & -5\sqrt{3} \\ 2\sqrt{3} & -3\sqrt{3}-1 \end{pmatrix}$$

a) [2 points] Find both complex eigenvalues of A.

b) [2 points] Find an eigenvector corresponding to each eigenvalue.

c) [3 points] Find an invertible matrix P and a rotation-scale matrix C such that $A = PCP^{-1}$.

d) [1 point] By what factor does C scale?

e) [2 points] By what angle does C rotate?

Problem 5.

In any given year, 10% of city dwellers will move to the country, while 90% will stay in the city. Likewise, 30% of country dwellers will move to the city, while 70% will stay in the country.

a) [3 points] Let x_n be the number of people in the city in year n, and let y_n be the number of people in the country in year n. Find a matrix A such that

$$A\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}.$$

b) [4 points] Compute the steady state of A.

c) [3 points] If the region (city plus country) starts off with 1,000 residents, about how many people will live in the city 100 years later (assuming the total population stays constant)?

[Scratch work]