MATH 1553-B
PRACTICE MIDTERM 2

Name

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Please read all instructions carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 60 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is not meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.
Suppose that $A$ is an $n \times n$ matrix that is not invertible. Let $T(x) = Ax$ be the linear transformation associated to $A$. Which of the following can you conclude? Circle all that apply.

a) $A$ has two identical columns

b) $\det(A) = 0$

c) $A$ has a row of zeros

d) There are two different vectors $u$ and $v$ in $\mathbb{R}^n$ with $T(u) = T(v)$

One more true-false problem:

e) T F If $A$ is a $2 \times 5$ matrix, then the solution set of $Ax = 0$ is a subspace of $\mathbb{R}^5$.
Problem 2.

Let

\[
A = \begin{pmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12
\end{pmatrix}
\quad \text{and} \quad
B = \begin{pmatrix}
5 & 6 & 7 & 8 \\
6 & 8 & 10 & 12 \\
-9 & -10 & -11 & -12
\end{pmatrix}.
\]

a) [1 point] What three row operations are needed to transform \(A\) into \(B\)?

\begin{align*}
\text{Operation 1:} & \\
\text{Operation 2:} & \\
\text{Operation 3:} & 
\end{align*}

b) [3 points] What are the elementary matrices for these three operations (in the same order)?

\begin{align*}
E_1 = & \\
E_2 = & \\
E_3 = & 
\end{align*}

c) [3 points] Write an equation for \(B\) in terms of \(A\) and \(E_1, E_2, E_3\).

\[B = \]

d) [3 points] Write an equation for \(A\) in terms of \(B\) and \(E_1, E_2, E_3\).

\[A = \]
Problem 3.

Let

\[ A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{pmatrix}. \]

Be sure to show your work in this problem.

a) [5 points] Find the inverse of \( A \), or explain why \( A \) is not invertible.

b) [3 points] Find the determinant of \( A \).

c) [2 points] Find the volume of the parallelepiped defined by the columns of \( A \).
Problem 4.

Consider the following matrix and its reduced row echelon form:

\[
A = \begin{pmatrix}
3 & 4 & 0 & 7 \\
1 & -5 & 2 & -2 \\
-1 & 4 & 0 & 3 \\
1 & -1 & 2 & 2
\end{pmatrix}
\sim
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]

a) [3 points] Find a basis for \( \text{Col}A \).

b) [3 points] Find a basis for \( \text{Nul}A \).

c) [2 points] What are rank\( A \) and dim \( \text{Nul}A \)?

d) [2 points] If \( B \) is any \( m \times n \) matrix, then

\[
\text{rank} B + \text{dim} \text{Nul} B =
\]
Problem 5.

Consider the vectors

\[ v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} \]

and the subspace

\[ V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \mid w = 0 \right\}. \]

a) [4 points] Explain why \( B_1 = \{ e_1, e_2, e_3 \} \) is a basis for \( V \).

b) [4 points] Explain why \( B_2 = \{ v_1, v_2, v_3 \} \) is a basis for \( V \).

c) [2 points] Find \( [v]_{B_2} \) if \( v = \begin{pmatrix} 4 \\ 4 \\ 4 \\ 0 \end{pmatrix} \).
Problem 6. [10 points]

Compute the determinant of the matrix

\[
\begin{pmatrix}
1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2
\end{pmatrix}^2.
\]
[Scratch work]