## Math 1553 Worksheet 10

November 11, 2016

1. Let $A$ and $B$ be $3 \times 3$ real matrices. Answer yes / no / maybe:
a) Can the eigenvalues of $A$ be 3,5 , and $2+i$ ?
b) Can $A$ have three complex (non-real) eigenvalues?
c) $\operatorname{Can} A$ have a complex eigenvalue with multiplicity 2 ?
d) Suppose that $A$ has one eigenvalue of algebraic multiplicity 3 . Is $A$ diagonalizable?
e) Suppose that $A$ has two distinct eigenvalues. Is it diagonalizable?
f) Suppose that $A$ has three distinct eigenvalues. Is it invertible?
g) If $A$ and $B$ both have eigenvalues $-1,0,1$, then $A$ is similar to $B$.
h) If $A$ and $B$ have the same eigenvalues, then $A$ is similar to $B$.
i) If $A$ and $B$ have one real and one complex eigenvalue in common, then $A$ is similar to $B$.
2. Let $A=\left(\begin{array}{rr}1 & 2 \\ -2 & 1\end{array}\right)$.
a) Find all (complex) eigenvalues and eigenvectors of $A$.
b) Write $A=P C P^{-1}$, where $C$ is a rotation followed by a scale.
c) What does $A$ do geometrically? Draw a picture.
3. Let $A=\left(\begin{array}{rrr}4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2\end{array}\right)$.
a) Find all (complex) eigenvalues and eigenvectors of $A$.
b) Write $A=P C P^{-1}$, where $C$ is a block diagonal matrix, as in the slides for $11 / 9$.
c) What does $A$ do geometrically? Draw a picture.
