Math 1553 Quiz 7
Solutions

1. [5 points] Write a mathematically correct definition of an eigenvector:

   “\(v\) is an eigenvector of an \(n \times n\) matrix \(A\) provided that
   
   \[v \neq 0\) and \(Av = \lambda v\) for some scalar \(\lambda\).”

2. [5 points] Find all eigenvalues of \(A\), and compute a basis for each eigenspace.

   \[
   A = \begin{pmatrix}
   1 & 2 \\
   0 & 3 
   \end{pmatrix}
   \]

   **Solution.**

   This is an upper-triangular matrix, so the eigenvalues are the diagonal entries 1 and 3. To find a basis for the 1-eigenspace, we compute

   \[
   A - I = \begin{pmatrix}
   0 & 2 \\
   0 & -2 
   \end{pmatrix}
   \sim_{\text{rref}} \begin{pmatrix}
   0 & 1 \\
   0 & 0 
   \end{pmatrix}.
   \]

   The parametric vector form for the general solution to \((A - I)v = 0\) is \(v = x(1\ 0)\), so a basis for the 1-eigenspace is \(\left\{ (1\ 0) \right\} \).

   To find a basis for the 3-eigenspace, we compute

   \[
   A - 3I = \begin{pmatrix}
   -2 & 2 \\
   0 & 0 
   \end{pmatrix}
   \sim_{\text{rref}} \begin{pmatrix}
   1 & -1 \\
   0 & 0 
   \end{pmatrix}.
   \]

   The parametric vector form for the general solution to \((A - 3I)v = 0\) is \(v = y(1\ 1)\), so a basis for the 3-eigenspace is \(\left\{ (1\ 1) \right\} \).