

Math 1553 Worksheet 5

September 23, 2016

Linear Transformations

1. Which of the following transformations are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by 32° in \mathbf{R}^2 .

b) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (z, x)$.

c) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

d) The matrix transformation with matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$.

e) The matrix transformation with matrix $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

2. Say A is an $m \times 2$ matrix. Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^m$ be the transformation defined by $T(x) = Ax$. If the columns of A are linearly independent, what does the image of T look like geometrically? What if they're linearly dependent?

3. For each matrix, describe what the associated matrix transformation does to \mathbf{R}^3 geometrically.

$$\text{a) } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{c) } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

4. The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, and $(1, 1, 1)$. The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the z -axis, and then projected onto the xy -plane.
- a) Express this transformation as a composition of two simpler transformations.
- b) Find a matrix C to represent the transformation that destroys the house. Express this as a product of the two matrices corresponding to the transformations in (a).