

### Math 1553 Quiz 3

#### Solutions

1. [5 points] Let  $v_1, v_2, \dots, v_p$  be vectors in  $\mathbf{R}^n$ . A vector  $b$  in  $\mathbf{R}^n$  is in  $\text{Span}\{v_1, v_2, \dots, v_p\}$  if and only if the matrix equation  $Ax = b$  has a solution, where  $A$  is what matrix?

#### Solution.

To say  $b$  is in the span of  $v_1, v_2, \dots, v_p$  means that there exist scalars  $x_1, x_2, \dots, x_p$  such that

$$x_1 v_1 + x_2 v_2 + \dots + x_p v_p = b.$$

This is the same as saying

$$\begin{pmatrix} | & | & \cdots & | \\ v_1 & v_2 & \cdots & v_p \\ | & | & \cdots & | \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{pmatrix} = b.$$

2. [1 point each] For which matrices  $A$  does the matrix equation  $Ax = b$  have a solution for *all*  $b$  in  $\mathbf{R}^2$ ?

a)  $\begin{pmatrix} 1 & 2 & 4 \\ -2 & -4 & -8 \end{pmatrix}$     b)  $\begin{pmatrix} 1 & 2 & 4 \\ -1 & -4 & -8 \end{pmatrix}$     c)  $\begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix}$     d)  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

#### Solution.

The equation  $Ax = b$  has a solution for all  $b$  in  $\mathbf{R}^2$  if and only if  $A$  has a pivot in each row. This can be checked by row reduction.

a)  $\begin{pmatrix} 1 & 2 & 4 \\ -2 & -4 & -8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{pmatrix}:$

does not have a pivot in each row.

b)  $\begin{pmatrix} 1 & 2 & 4 \\ -1 & -4 & -8 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & -2 & -4 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}:$

does have a pivot in each row.

c)  $\begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}:$

does not have a pivot in each row.

d)  $\begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}:$

does not have a pivot in each row. (It can't because it's too skinny.)