1. [1 point each] Consider the following picture of two vectors \( v, w \):

For each of the labeled points, estimate the coefficients \( x, y \) such that the linear combination \( xv + yw \) is the vector ending at that point. (Use the parallelogram law for vector addition; you needn't show your work.)

\[
\begin{align*}
_____ v + _____ w &= a \\
_____ v + _____ w &= b \\
_____ v + _____ w &= c \\
_____ v + _____ w &= d \\
_____ v + _____ w &= e \\
\end{align*}
\]

Solution.

As you can tell from the grid, you reach \( a \) by following \( v \) once then \( w \) twice. Hence \( a = v + 2w \). Similarly, \( b = -\frac{1}{2}v - w \), \( c = -v + 0w \), \( d = \frac{1}{2}v - \frac{3}{2}w \), and \( e = \frac{3}{4}v + \frac{3}{4}w \).
2. [2 points each] Which of the following vectors are in the span of \( \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \) and \( \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} \)?

\[
\begin{align*}
\text{a)} & \quad \begin{pmatrix} -14 \\ -10 \\ 11 \end{pmatrix} \\
\text{b)} & \quad \begin{pmatrix} -14 \\ -10 \\ -11 \end{pmatrix} \\
\text{c)} & \quad \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\end{align*}
\]

(Show at least a bit of your work.)

**Solution.**

a) Deciding if \( \begin{pmatrix} -14 \\ -10 \\ 11 \end{pmatrix} \) is a linear combination of \( \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \) and \( \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} \) amounts to row reducing the augmented matrix:

\[
\begin{pmatrix}
1 & 6 & -14 \\
-1 & 2 & -10 \\
2 & -1 & 11
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 6 & -14 \\
0 & 1 & 3 \\
0 & 0 & 0
\end{pmatrix}
\]

We can stop here, because we can already see that the system is consistent. (You weren’t asked to find the coefficients in a linear combination.)

b) As before, we row reduce

\[
\begin{pmatrix}
1 & 6 & -14 \\
-1 & 2 & -10 \\
2 & -1 & -11
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 6 & -14 \\
0 & 1 & 3 \\
0 & 0 & -22
\end{pmatrix}
\]

We can stop here, because this corresponds to an inconsistent linear system (the last equation being \( 0 = -22 \)).

c) The zero vector is in any span:

\[
\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 0 \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}.
\]