Math 1553 Quiz 1

Solutions

1. [5 points] Put the following matrix into reduced row echelon form using elementary row operations.

$$\begin{pmatrix} 1 & 3 & -2 & 4 \\ 2 & 6 & -2 & 6 \\ 5 & 15 & -8 & 22 \end{pmatrix}$$

Solution.

$$\begin{pmatrix} 1 & 3 & -2 & 4 \\ 2 & 6 & -2 & 6 \\ 5 & 15 & -8 & 22 \end{pmatrix} \xrightarrow{\text{2nd}-2\times\text{1st}} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 2 & -2 \\ 5 & 15 & -8 & 22 \end{pmatrix}$$

$$\xrightarrow{\text{3rd}-5\times\text{1st}} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{2nd}/2} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

$$\xrightarrow{\text{3rd}-2\times\text{2nd}} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\xrightarrow{\text{3rd}/4} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{2nd}+\text{3rd}} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{1st}+2\times\text{2nd}} \begin{pmatrix} 1 & 3 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{1st}-4\times\text{3rd}} \boxed{\begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

2. [2 points each] Determine whether the linear system corresponding to each of the following augmented matrices has no solutions, a unique solution, or infinitely many solutions.

a) $\left(\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 2 & 6 & -2 & 6 \\ 5 & 15 & -8 & 22 \end{array}\right)$ **b)** $\left(\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ **c)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

Solution.

a) No solutions. (This is the same matrix as in problem 1, so you already did the row reduction.)

b) Infinitely many solutions. (The second variable is free, and it is consistent.)

c) Unique solution: $x = 4$, $y = -1$, $z = 2$ (and $0 = 0$).