Math 4107 Homework 4

Due at the beginning of class on Wednesday, February 10.

Exercises from the text (justify your answers!):

§2.7 #1
§2.8 #1, 2, 3, 6, 9, 10, 11
§2.9 #3

Remark. Exercise 2.9.3 implies that a number n is divisible by 9 if and only if the sum m of its decimal digits is divisible by 9. Of course, this number m is likewise divisible by 9 if and only if the sum of *its* digits is divisible by 9. Repeating gives a nice algorithm for deciding if a number is divisible by 9. For example, 1805301 is divisible by 9 because $1+8+0+5+3+0+1 = 18$ is divisible by 9 because $1+8 = 9$ is divisible by 9.

This also works with 3 replacing 9. Do you see why? Does this work for any other numbers?

Exercise not from the text:

1. Let G be a group and let $H \leq G$ be a subgroup. Find a bijection

$$\{\text{left cosets of } H \text{ in } G\} \longleftrightarrow \{\text{right cosets of } H \text{ in } G\}.$$

You may not assume that G, H, or $[G : H]$ is finite. [Hint: cf. Exercise 2.6.9.]

It follows that the index $[G : H]$ may be defined using either right cosets or left cosets, even when G and H are infinite.