

Math 4108 Homework 13

Due at the beginning of class on Tuesday, April 14.

§13.5 #1, 2

§13.6 #1, 2, 3

Not from the text:

1. Let $\delta = \sqrt{-5}$ and $R = \mathbf{Z}[\delta]$. Let $A \subset R$ be a nonzero ideal and let n be the positive integer such that $\overline{A}A = (n)$.
 - (a) Find n when $A = (\alpha)$ is a principal ideal.
 - (b) Find n when $A = (\alpha, \frac{1}{2}\alpha(1 + \delta))$ is not a principal ideal (cf. Theorem 3.3.3).
2. Let R be a quadratic integer ring.
 - (a) If $A \subset R$ is a nonzero ideal, prove that $A \cap \mathbf{Z}$ is an ideal in \mathbf{Z} but not in R .
 - (b) Let P be a nonzero prime ideal of R . Prove that $P \cap \mathbf{Z}$ is a prime ideal. Hence $P \cap \mathbf{Z} = p\mathbf{Z}$ for $p \in \mathbf{Z}$ a prime number.
 - (c) In the situation of (b), what is the relationship between P and $(p) = pR$? [The answer depends on whether or not p remains prime in R .]

Note:

In #13.5.1, just use the results from §13.6.