The construction of orthogonal wavelet bases of order 2 of compactly supported wavelets is now well understood. For the construction of orthonormal bases of compactly supported wavelets, see [99, 100].

The paper is based on joint work with A. Cohen and P. Vial.

USA

Murray Hill, NJ 07974

AT&T Bell Laboratories

Friedland Daubechies
\[ \text{for } x \in \mathbb{R}, \quad (x)_{0} \phi \text{ is an orthonormal basis for } L^{2} \]
The text is not legible due to the image quality but appears to be discussing wavelet theory and possibly the decomposition and reconstruction of functions. The content involves mathematical notation and theoretical concepts related to wavelets and their properties.
more details, see Cohen, Daubechies and Vial (1992).

One can however characterize \( f \in L^2 \) if one uses two pairs of orthonormal bases. Define

\begin{align*}
\phi(t) &= \begin{cases} t, & t \leq 1, \\
0, & t > 1.
\end{cases}
\end{align*}

and the \( \psi \) pair fails. The \( \phi \) \( \psi \) pair can then be characterized with \( f = \langle f, \phi \rangle \) for all \( f \in L^2 \), which is only possible if the integral

\begin{align*}
\int_{-\infty}^{\infty} f(t) \phi(t) \, dt
\end{align*}

smoothness of \( \phi \). If one does to see what goes wrong if \( f = 0 \) for all \( f \in L^2 \), the part from the only \( \phi \) \( \psi \) pair that must be replaced by a \( \mathcal{Z} \)-bounded-type space

\begin{align*}
\|f\|_{\mathcal{Z}}^2 &= \int_{-\infty}^{\infty} |f(t)|^2 \, dt
\end{align*}

is usual.)

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\end{align*}

for \( f = 1 \) a similar result holds with \( \mathcal{C} \) replaced by a \( \mathcal{Z} \)-bounded-type space.
\[ N > j, \gamma > 1 + N - (\gamma + x) \phi (\gamma + x) x p_0 \]

computed explicitly. This involves the computation of integrals of the type

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in order to implement the scheme. All the outermost integration and projection matrices have to be

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in such an image analysis work, which usually are typically squares with 256x256 or 512x512 pixels. In

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number of scaling functions is not a power of 2, therefore it is a nuisance for practical applications.

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essential that these two families have the same number of coefficients at every scale. Then the

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coefficients get split as well as scaling coefficients, using the same filters, and for this is

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be generalized to wavelet packets on the interval in a wavelet packet construction, wavelet

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functions at resolution \( j \) larger than the number of wavelets. However, this construction cannot

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where \( p \leq \) the regularity of the original wavelet bases, \( \psi \in C^p \). Because the number of scaling

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ensures that they are orthonormal bases for the Hilbert space, and vanishing moment properties

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functions on \( [0, 1] \) at the coarsest scale under consideration, these scaled wavelets constitute an

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and the same regularity as the original \( \psi \), together with an orthonormal family of wavelet bases

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of the reconstruction is an orthonormal family of wavelets in \( L^2 [0, 1] \), which more than half the support is within \( [0, 1] \). For details, see Meyer [1992]. In contrast, the result

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functions have to be constructed explicitly. In particular, the local edge functions are obtained by

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defined on the whole line (which happen to have their support contained in \([0, 1]\)). The local edge functions are simply those \( \phi - \phi \) or \( \psi - \psi \) whose support is contained in \([0, 1]\). The local

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is larger, \( \gamma^2 + 2N - 2 \). The \( \psi \) functions are simply those \( \phi - \phi \) or \( \psi - \psi \) whose support is

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the local number of wavelets at scale \( j \), plus \( 2j \), but the local number of scaling functions

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are obtained by orthogonalizing the \( \psi \) functions and \( \phi \). These \( \psi \) functions, \( \phi \), and the

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are generated by \( \lambda \) such a multi-resolution analysis. The \( \psi \) functions are the wavelet

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and one of the compactly supported bases in Daubechies [1988]11, with vanishing moments.

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A fourth solution was proposed in Meyer [1992]. The starting point of this construction is

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A different construction of integral wavelets.

The construction was made independently by P. L. Lions and B. Jourdain. After completing our work, we learned that a similar construction was given independently by P. L. Lions and B. Jourdain.

The construction follows that all the corresponding wavelets at the edge are well as in the interior, have a finite set of degrees, and this is sufficient to ensure that we have an infinite set of wavelet bases.

We can be written as linear combinations of the scaling functions at every resolution level. Moreover, as in Meyer's case, all the polynomials on the union of a finite number are exactly equal at resolution I = 1.

The wavelet square functions violate certain properties, leading them so that the total number is exactly 2^I. However, as in Meyer's construction, we can use "interpolation" and "extractor" scaling functions at every resolution.

This paper presents a thin solution, also derived from compactly supported wavelet basis for the edge of the interval [0, 1]. This is the reason why B. Jourdain, in an application involving such extraction operations to problems in collaboration with B. Dubremetz, decided to develop a construction different from Meyer's. Another instance where this model has been mentioned is in the papers of Meyer, where one can count the number of polynomials on the edge.

The expansion of those edge scaling functions that are obtained from restricting \( \phi \) to \([0, 1]\) which have only a tiny piece of their support in \([0, 1]\) can have a huge amplitude outside of \([0, 1]\). This does not work so well in practice.

Any smooth function \( f \) extending \( f \) to \([0, +\infty)\) can be written as linear combinations of wavelet bases of the form

\[
\sum_{n} \psi_{n} \cdot f(x)
\]

where \( \psi_{n} \) is a wavelet basis function and \( f(x) \) is the function to be approximated. The resulting system is however very badly conditioned, and the solution can be computed by solving an

\[
(1 - N + x) \phi \cdot x^{p} \sum_{i, j} \phi (2i + N - x) \phi \cdot x^{p} \]

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to check is that by adjective functions in this way we don't lose the framework of a
shows, incidentally, that $\phi$ is orthogonal to all the interior $\phi$. The only thing that we have
the compact support. It also
$$ (y - x)^{\infty - K_{\infty}} (y - x) $$
where $y$ and the interior $\phi$. The interior $\phi$ and this edge function $\phi$ together ensure all
the consistence on $[0, \infty)$. Moreover, because $\phi = 0$ and $y$ do not even generate the consistence on $[0, \infty)$, at least from the inside of $\phi$, they are supported on $[0, \infty)$. By the same idea, the interior $\phi$ are
the half space $[0, \infty, \infty)$ instead of on $[0, \infty)$. Here then only have to deal with the half edge, and
on the halp. The $[0, \infty, \infty)$ up to a certain degree. Let us illustrate the principle of the construction of working
adapted edge scaling functions in such a way that their union still generates all polynomials.
Our goal is to retain the interior scaling functions, and to add
$$ \{ N^{N - 1} \} = \phi $$
vaarant (see [Dautovskevic, 1992]). We choose to translate them so that support
Our starting point is again the $N$ vanishing moment family of Dautovskevic (1988) or a
related construction, from the other point of view, is in Heyer, Kovačević, and

$$ L = N $$
for the interior edge in the construction of $L$. We may

\[ \text{Figure 1: The adapted scaling functions in } \mathcal{F}(L) \]
\[
(1 + N - u + x)\phi \left( \frac{y}{u} \right) \sum_{\gamma = u}^{\phi} = (x)_{\phi}^y
\]

This is essentially all there is to the construction we propose here. If we want the edge functions & edge solutions hierarchy of nested spaces.

Similar inclusions hold immediately if we scale by other integer powers of 2, and we still have

\[
\left( \frac{1}{w} \right) A = \left\{ 1 - N < \gamma \leq \frac{1}{w} \right\} \supset \left\{ 1 - N < \gamma \right\} \supset \left\{ 1 - N < \gamma \right\}
\]

follows therefore that

\[
\left( \frac{1}{w} \right) A = \left\{ 1 - N < \gamma \leq \frac{1}{w} \right\}
\]

and where we have used that \( \gamma \geq 0 \) for \( u = \gamma \)

\[
\left( x - z \right) \phi \left( \frac{y}{u} \right) \sum_{\gamma = u}^{\phi} = (x)_{\phi}^y
\]

and

\[
\left( x - z \right) \phi \left( \frac{y}{u} \right) \sum_{\gamma = u}^{\phi} = (x - z)\phi
\]

multiresolution hierarchy. We have however.
\[
\sum_{q=0}^{N} \phi(x) \phi(x) + \sum_{q=0}^{N} \phi(x) \phi(x) + \sum_{q=0}^{N} \phi(x) \phi(x) = \sum_{q=0}^{N} \phi(x) \phi(x)
\]

The orthonormalization procedure (such that
and
existing constants and \( H_q \)) can be computed explicitly from the \( b_q \) in (2) (and
thereby a recursion relation similar to (2) and inherited by all the scales). Explicitly, there.

The orthonormal \( \phi_i \) constructed with staggered supports along the lines indicated above,

higher values of \( k \),

leading to an explicit formula for \( (\phi_i, \phi_j) \), since it is known, the proceeds similarly

for \( k = 0 \). For instance, we have

overlap matrix, we use the recurrence (2). To compute this
orthonormalization explicitly, we need again the overlap matrix \( (\phi_i, \phi_j) \). To carry out the Gram-Schmidt
it will have staggered support: support \( i \), \( i = 0, \cdots, N \).

If we work down to lower values of \( k \), then the resulting orthonormal \( \phi_i \) are
already orthonormal to the orthonormal \( \phi_i \) scaling then leads to an orthonormal basis for

One can obtain an orthonormal basis for \( \phi_i \) by orthonormalizing the \( \phi_i \), since they are

\[
\sum_{q=0}^{N} \phi(x) \phi(x) + \sum_{q=0}^{N} \phi(x) \phi(x) + \sum_{q=0}^{N} \phi(x) \phi(x) = \sum_{q=0}^{N} \phi(x) \phi(x)
\]

For all products, see Cohen, Daubechies and Vial (1992).

(2)

\[
(w - x) \phi(x) + \sum_{q=0}^{N} \phi(x) \phi(x) + \sum_{q=0}^{N} \phi(x) \phi(x) = \sum_{q=0}^{N} \phi(x) \phi(x)
\]

there exist constants \( a_q \) which can be computed explicitly, so that
the \( w \) is in \( N \), they generate all the polynomials up to degree \( \infty \). Finally, \( N \geq w \) and \( N \geq w \), they are independent, and orthonormal to the \( \phi_i \), together with

These are all completely supported, and their supports are staggered, i.e., support

103
We have also have the f + w N+1, which we now denote as the "shifted coefficient" in addition to the \( w_n \). In practical applications, this is really needed as the shift coefficients in addition to the whole time we have no explicit analytic expression for the wavelets and scaling functions on the interval. Note that, like on the 0 interval, the scaling function for \( N = 4 \) at the left end of \([0, \infty)\). However, the results are the same here to be repeated.

\[
(\varepsilon_{1+\varepsilon_2} f) \circ \mathcal{S} \leq |(\varepsilon_{1+\varepsilon_2} f(x))| |(\varepsilon_{1+\varepsilon_2} f(x))|
\]

This completes our explicit construction at least at a local end. The same has to be repeated

\[
\langle \mathcal{S} \rangle_{0} = \int_{0}^{1} \mathcal{S} \quad \text{so that}
\]

Moreover, there exists a constant \( C \) such that

\[
\langle \mathcal{S} \rangle_{0} \leq C
\]

In a final step, these \( \phi_{k} \) can be orthonormalized and we end up with an orthonormal family

\[
\langle \mathcal{S} \rangle_{0} = \int_{0}^{1} \mathcal{S} \quad \text{so that}
\]

The recursion relation (4) can be written in the form of the orthonormal functions in \( \mathcal{S} \). Hence of

\[
\langle \mathcal{S} \rangle_{0} = \int_{0}^{1} \mathcal{S} \quad \text{so that}
\]

Define 1 the \( \phi_{k} \) and any \( \phi_{k} \) belong to \( \mathcal{S} \) all \( \phi_{k} \) as a linear combination of \( \phi_{k} \) in \( \mathcal{S} \). The orthonormal functions in \( \mathcal{S} \) and we are looking for extra functions in \( \mathcal{S} \).

For simplicity we denote \( \mathcal{S} \) to be the whole interval. In the other hand it is easy to check that the extra \( 2N \) functions \( \phi_{k} \) are in \( \mathcal{S} \).

1 \( \mathcal{S} \) and \( \phi_{k} \) belong to \( \mathcal{S} \) and we denote them. However, we denote them. We denote them. We denote them.

Since they are all orthonormal, we denote them. The whole interval. In the other hand it is easy to check that the extra 2N functions \( \phi_{k} \) are in \( \mathcal{S} \).

From dimension counting, it immediately follows that the dimension of \( \mathcal{S} \) is the 2N. Therefore, we denote them. We denote them. We denote them.
We have assumed that we want the scaling functions to generate all possible polynomials up to a certain degree. If the internal wavelets are used to solve a differential equation, then it may be useful to adapt the construction so that all the scaling functions and wavelets involved satisfy the initial conditions.

Many wavelets are possible. One can, for instance, start from completely different families of wavelets with a different number of vanishing moments, and adapt the number of additional wavelets to the degree of the problem. We can adapt the scaling functions to be polynomials themselves. Contrast the adapted scaling functions $\phi_{j,0}$ at the left edge in our new construction for $N = 4$ with those at the right edge in the old construction for $N = 1$. The adapted scaling functions in these plots are less oscillatory than those in [266].

![Diagram of scaling functions](image)

In §4, on [0,1], the functions $\phi_{j,0} = 0$ are pure polynomials of degree $N - 1$. Tables for these filter coefficients can be found in Cohen, Daubechies, and Vial [1] (1992).
Figure 3: Different line frequency filters.

Part from the obvious applications mentioned above (image analysis, solving PDEs with numerated stability). The construction by P. C. Loewenstein, which is essentially the same
References

In continuous wavelet transforms of data confined to a finite interval, such extensions could be used to avoid boundary problems. For the natural extension of $\psi^{-\frac{a}{2}}$ (where every $\psi^{-\frac{a}{2}}$ belongs to the wavelet packet space), this loss is work in progress. Another possible application of wavelets on the interval is the extension problem. The extension should be proportional to the support of the different mother wavelets. A straightforward application of wavelet packets derived from the cone-consecutive intervals. A straightforward extension of wavelet packets derived from the cone-consecutive intervals. A straightforward extension of wavelet packets derived from the cone-consecutive intervals.