1. Show that a holomorphic bijection between Riemann surfaces is a biholomorphism.

2. Show that if $f : X \to Y$ is a non-constant holomorphic map between Riemann surfaces, then the inverse image of each $y \in Y$ is a closed, discrete subset of $X$.

3. Suppose that $p : X \to Y$ is a covering map. Show that if $Y$ is a Riemann surface, then $X$ has a unique complex structure such that $p$ is holomorphic. Note that, in particular, the universal covering of every Riemann surface is naturally a Riemann surface. Show that if $Z$ is a Riemann surface, then a function $f : Z \to X$ is holomorphic if and only if $p \circ f : Z \to Y$ is holomorphic.

4. Suppose that $p : X \to Y$ is a Galois (i.e., normal or regular) covering map with Galois group (i.e., group of deck transformations) $G$. Suppose that $X$ is a Riemann surface and that $G$ acts on $X$ as a group of biholomorphisms. Show that $Y$ has a unique complex structure such that $p$ is holomorphic.

5. Suppose that $U$ is an open neighbourhood of 0 in $\mathbb{C}$ and that $\phi : U \to U$ is a biholomorphism of order $n$ that fixes the origin. Show that there is a local holomorphic coordinate $w$ in $\mathbb{C}$ centered at 0 and an $\epsilon > 0$ such that
   
   (i) $D(0, \epsilon) \subset U$,
   (ii) $\phi(D) \subseteq D$, where $D = D(0, \epsilon)$,
   (iii) $\phi(w) = \zeta_n w$, where $\zeta_n = e^{2\pi i/n}$.

   Hint: show that the function $U \to \mathbb{C}$ defined by $z \mapsto \prod_{j=0}^{n-1} \phi^j(z)$ has a holomorphic $n$th root in a neighbourhood of 0. Take $w$ to be an $n$th root.

6. Suppose that $X$ is a Riemann surface and that $G$ is a finite subgroup of $\text{Aut} X$. Denote the stabilizer of $P \in X$ by $G_P$.

   (i) Show that each $P \in X$ has a $G_P$-invariant coordinate neighbourhood.
   (ii) Show that the stabilizer of each $P \in X$ is cyclic.
(iii) Show that the quotient $G\backslash X$ of $X$ by $G$ has a natural complex structure such that the projection $X \to G\backslash X$ is holomorphic.