Math 611
Problem Set 5

Due: Thursday December 1, 2022

1. Suppose that $Y$ is a compact orientable surface of genus $g$. Suppose that $D_1, \ldots, D_n$ are $n > 0$ disjoint closed disks embedded in $Y$. Let

$$X = Y - \bigcup_{j=1}^{n} \hat{D}_j.$$ 

That is, $X$ is a genus $g$ surface with $n$ boundary components, each of which is a circle. Compute

$$H_\bullet(X; \mathbb{R})$$

and

$$H_\bullet(X, \partial X; \mathbb{R})$$

where $\partial X$ denotes the boundary of $X$, which is the disjoint union of $n$ circles.

2. Let $X$ be a compact orientable surface of type described in the previous in the previous problem. Let $Z$ be the surface obtained by attaching $n$ Möbius bands to $X$, one to each of its boundary components. Compute the integral and $\mathbb{F}_2$ homology of $Z$. 