1. Suppose that $p : Y \to X$ is a covering map and that $\Delta$ is a contractible subset of $X$. Show that any two distinct lifts of $\Delta$ to $Y$ are disjoint.

2. A solid torus is a topological space that is homeomorphic to $S^1 \times B^2$. Consider $S^3$ to be the unit sphere in $\mathbb{C}^2$:

$$S^3 = \{(x, y) \in \mathbb{C}^2 : |x|^2 + |y|^2 = 1\}.$$ 

Fix a real number $a$ satisfying $0 < a < 1$. Let $T(a) = \{(x, y) \in S^3 : |x|^2 = a\}$, $U_1(a) = \{(x, y) \in S^3 : |x|^2 \leq a\}$ and $U_2(a) = \{(x, y) \in S^3 : |x|^2 \geq a\}$. Show that $T(a)$ is a 2-torus and that $U_1(a)$ and $U_2(a)$ are solid tori that intersect in $T(a)$. Deduce that $S^3$ is homeomorphic to

$$U_1(a) \cup T(a) \cup U_2(a) := (U_1(a) \amalg U_2(a)) / \sim$$

where the equivalence relation $\sim$ identifies $x \in U_1(a)$ with $y \in U_2(a)$ if and only if $x = y \in T(a)$.

3. As above, view $S^3$ as the unit sphere in $\mathbb{C}^2$. Let $L_1 = \{(x, y) \in S^3 : x = 0\}$ and $L_2 = \{(x, y) \in S^3 : y = 0\}$. Show that $L_1$ and $L_2$ are disjoint imbedded circles in $S^3$. Show that $T(a)$ is a deformation retract of $S^3 - (L_1 \cup L_2)$. Use this to show that $\pi_1(S^3 - (L_1 \cup L_2), x_0)$ is isomorphic to $\mathbb{Z}^2$.

4. Suppose that $m$ and $n$ are positive integers. Denote the affine curve $x^m = y^n$ in $\mathbb{C}^2$ by $C$. Set $L = C \cap S^3$. For $\lambda \in \mathbb{C}$, define

$$\lambda \cdot (x, y) = (\lambda^n x, \lambda^m y)$$

(i) Show that the function $f : \mathbb{R}_{\geq 0} \times S^3 \to \mathbb{C}^2$ defined by $f(t, \xi) = t \cdot \xi$ induces a homeomorphism

$$(\text{cone}(S^3), \text{cone} L, 0) \to (\mathbb{C}^2, C, 0)$$

and a homeomorphism

$$\mathbb{R}_{\geq 0} \times (S^3 - L) \to \mathbb{C}^2 - C.$$
(ii) Show that $L$ is imbedded in $S^3$ as
\[ \{ (\theta, \phi) \in S^1 \times S^1 : m\theta \equiv n\phi \mod 2\pi \} \]
where $S^1 \times S^1$ denotes the torus
\[ \{ (x, y) \in S^3 : |x|^2 = a \text{ and } |y|^2 = 1 - a \} \]
for some suitable $a$ satisfying $0 < a < 1$. (We say that $L$ is a torus link of type $(m, n)$.)

(iii) Show that the number of connected components of $L$ is the greatest common divisor of $m$ and $n$.

(iv) Suppose that $x \in S^3 - L$. Show that the inclusion $j : S^3 - L \hookrightarrow \mathbb{C}^2 - C$ induces an isomorphism
\[ j_* : \pi_1(S^3 - L, x) \rightarrow \pi_1(\mathbb{C}^2 - C, x) \].