1. Expand
\[ f(z) = \frac{1}{z^3} \]
as a power series about \( z = -2 \). What is the radius of convergence of this power series?

2. For what integers \( n \geq 0 \) (if any) is
\[ \frac{z^n}{z - \sin z} \]
analytic at \( z = 0 \)?

3. (i) Show that \( \cos z = 0 \) if and only if \( z \) is an odd multiple of \( \pi/2 \).
(ii) Compute
\[ \int_{|z-z_0|=1} \frac{\cos z}{z - \pi} \, dz \]
when (a) \( z_0 = 0 \); (b) \( z_0 = \pi \), and (c) \( z_0 = 3 \).

4. Suppose that \( S \) is an open set that contains the closed unit disk \( |z| \leq 1 \). Show that if \( f : S \to \mathbb{C} \) an analytic function that maps the unit disk into itself and if \( f(z) = \sum_{n=0}^{\infty} c_n z^n \), then \( |c_n| \leq 1 \) for all \( n \). Hint: use the formula for \( c_n \).

5. (i) Let \( S \) be the upper half plane \( \text{Im}(z) > 0 \). Show that the exponential function \( z \mapsto e^{iz} \) maps \( S \) into the unit disk.
(ii) Show that if \( f \) is an entire function whose image lies in any half plane, then \( f \) is constant. Hint: Use the first part and Liouville’s Theorem.