

TABLEAU COMPLEXES

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ABSTRACT. Let X, Y be finite sets and T a set of functions from $X \rightarrow Y$ which we will call “tableaux”. We define a simplicial complex whose facets, all of the same dimension, correspond to these tableaux. Such *tableau complexes* have many nice properties, and are frequently homeomorphic to balls, which we prove using vertex decompositions [BP79].

In our motivating example, the facets are labeled by semistandard Young tableaux, and the more general interior faces are labeled by Buch’s set-valued semistandard tableaux. One vertex decomposition of this “Young tableau complex” parallels Lascoux’s transition formula for vexillary double Grothendieck polynomials [La01, La03]. Consequently, we obtain formulae (both old and new) for these polynomials. In particular, we present a common generalization of the formulae of Wachs [Wa85] and Buch [Bu02], each of which implies the classical tableau formula for Schur polynomials.

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