Topological brain statistics

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joint with

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Summer STEM Academy
School of Medicine Office of Clinical Research
Duke University

20 June 2019
Goal: Statistical analysis taking 3D geometry into account

- predict stroke tendency
- screen for loci of pathology, such as tumors
- explore how age affects blood vessels
Magnetic Resonance Angiography (MRA)

from Elizabeth Bullitt, Dept. of Neurosurgery, UNC-CH
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Tube tracking

[Bullitt and Aylward, 2002]
Tube tracking

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Brain artery trees

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The data structure:
Topological space $X \rightsquigarrow$ homology $H_iX$ for each dimension $i$.

- set of “$i$-dimensional holes” in $X$
Homology

Topological space $X \leadsto$ homology $H_i X$ for each dimension $i$.

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$\# H_1 = 1$
Homology

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![Diagram showing a circle and a sphere with points to illustrate $H_1$]

$\#H_1 = 1$
Topological space $X \rightsquigarrow$ homology $H_iX$ for each dimension $i$.

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\[ \#H_1 = 1 \quad \#H_1 = 0 \]
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![Diagram showing topological spaces and their homology]

- $\# H_1 = 1$
- $\# H_1 = 0$
- $\# H_2 = 1$
Homology

Topological space $X \xrightarrow{\sim} \text{homology } H_i X$ for each dimension $i$.

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$\# H_1 = 1$
$\# H_1 = 0$
$\# H_2 = 1$
$\# H_1 = 2$
Topological space $X \rightsquigarrow$ homology $H_i X$ for each dimension $i$.

- set of “$i$-dimensional holes” in $X$

\[
\begin{align*}
\#H_1 &= 1 \\
\#H_1 &= 0 \\
\#H_2 &= 1 \\
\#H_1 &= 2 \\
\#H_2 &= 1
\end{align*}
\]
Topological space $X \leadsto$ homology $H_i X$ for each dimension $i$.

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\#H_1 &= 0 \\
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- $i = 0$ case: $H_0$ is the set of connected components of $X$
Topological space $X \rightsquigarrow$ homology $H_i X$ for each dimension $i$.

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$\begin{align*}
\# H_0 &= 1 \\
\# H_1 &= 1 \\
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\end{align*}$

- $i = 0$ case: $H_0$ is the set of connected components of $X$
Homology

Topological space $X$ maps to homology $H_i X$ for each dimension $i$.

- set of “$i$-dimensional holes” in $X$

\[
\begin{align*}
\# H_0 &= 1 \\
\# H_1 &= 1 \\
\# H_2 &= 1
\end{align*}
\]

- $i = 0$ case: $H_0$ is the set of connected components of $X$

Formally: homology $H_i X$ is a vector space for each dimension $i$. 
Persistent homology

Build $X$ step by step

- measure evolving topology.

**Def.** Suppose $X$ has increasing subspaces $\emptyset = X_0 \subset X_1 \subset \cdots \subset X_m = X$

- persistent homology, a sequence of sets: $H_iX_1 \to H_iX_2 \to \cdots \to H_iX_m$

- a feature persists from $j$ to $k$ if it appears first in $H_iX_j$ and last in $H_iX_k$

**Example:** For $X$ in 3D, let $X_k =$ the part of $X$ below a plane $H_k$

**History.** invented by [Frosini, Landi 1999], [Robins 1999], [Edelsbrunner, Letscher, Zomorodian 2002]: includes efficient computation; [Carlsson, Scolamiero, Turner, many others]: additional theory, applications
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Example: expanding balls
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\[ \#H_0 = 31 \]
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Example: expanding balls

\[ \#H_0 = 26 \]
Example: expanding balls

$\#H_0 = 21$
Example: expanding balls

\[ \#H_0 = 12 \]
Example: expanding balls

$\# H_0 = 6$
Example: expanding balls

\[ \#H_0 = 2 \]
Example: expanding balls

$\#H_0 = 2$
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$\# H_0 = 1 \quad \# H_1 = 2$
Example: expanding balls

\#H_0 = 1 \quad \#H_1 = 1
Example: expanding balls

\[ \#H_0 = 1 \quad \#H_1 = 1 \]
Example: expanding balls

\[ \#H_0 = 1 \quad \#H_1 = 3 \]
Example: expanding balls

\[ \#H_0 = 1 \quad \#H_1 = 1 \]
Example: expanding balls

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Bar codes

Data structure: 3D tree $\leadsto$ bar code

- vertical line segments $[t, t']$ (each plotted at $x$-coordinate $t$)
- one for each feature with birth time $t$ and death time $t'$. 
Bar codes

Data structure: 3D tree $\rightsquigarrow$ bar code

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Diagrams, no inf or short (< 0.1) lengths, Case 71, Age = 32, Sex = F, Hand = R
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Sweeping geometry

Goal: statistical analysis taking into account

- 3D structure, in particular
- “bendiness”, or “tortuosity”
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Sweep across with a plane:
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Sweep across with a plane:

Record:
- birth time of each new component
- death of each component (when it joins to an older component)
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Sweep across with a plane:

Record:
- birth time of each new component
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Sweeping geometry

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![Graph showing sweeping geometry]

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Statistics

Reduce to linear methods

- 3D tree $\leadsto$ bar code $\leadsto$ vector in $\mathbb{R}^{100}$: top 100 bar lengths, in decreasing order
- correlate with age

Conclusions [with Bendich, Marron, Pieloch, Skwerer 2016]
Longest bars in older brains tend to be shorter and later.

Similar results after accounting for
- natural variation in overall brain size or
- known correlation of age vs. total vessel length [Bullitt, et al. 2005].

Moral
Topology can detect statistically significant geometric motifs.
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Lesson for students
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Thank You