Multiplier ideals of sums via cellular resolutions

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Report on joint work with Shin-Yao Jow (Michigan)

Outline

Multiplier ideals

Fix:

- smooth variety X over $\mathbb{C}$,
- ideal sheaf $\mathfrak{a} \subseteq \mathcal{O}_X$
- "weighting coefficient" $\alpha \in \mathbb{R}_{\geq 0}$

Def: Multiplier ideal $\mathcal{J}(X, \mathfrak{a}^\alpha) = \mathcal{J}(\mathfrak{a}^\alpha) \subseteq \mathcal{O}_X$:
$f_1, \ldots, f_\ell$ local generators of $\mathfrak{a} \Rightarrow$

$$\mathcal{J}(\mathfrak{a}^\alpha) \overset{\text{locally}}{=} \left\{ h \in \mathcal{O}_X \;\middle|\; \frac{|h|^2}{(|f_1|^2 + \cdots + |f_\ell|^2)^\alpha} \text{ is locally integrable} \right\}.$$

Idea: general $f \in \mathfrak{a}$ vanishes to high order
$\Rightarrow$ need h to vanish sufficiently

- measures singularities common to all $f \in \mathfrak{a}$:
  worse singularity of $\mathfrak{a} \leftrightarrow$ deeper $\mathcal{J}(\mathfrak{a}^\alpha)$
- applications: birational geometry, positivity, asymptotics, ...

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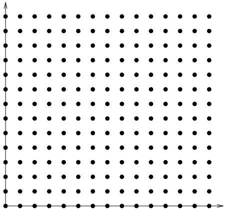

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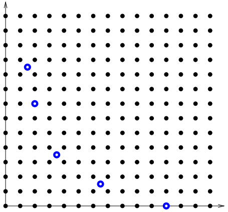

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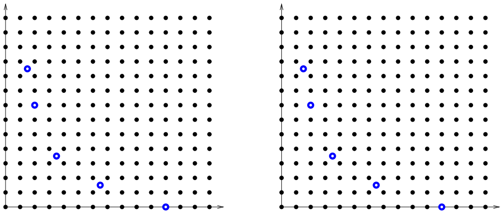

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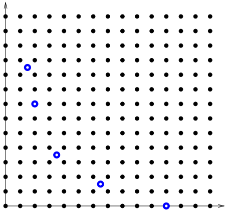

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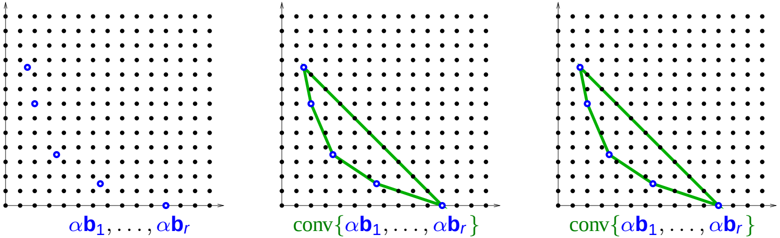

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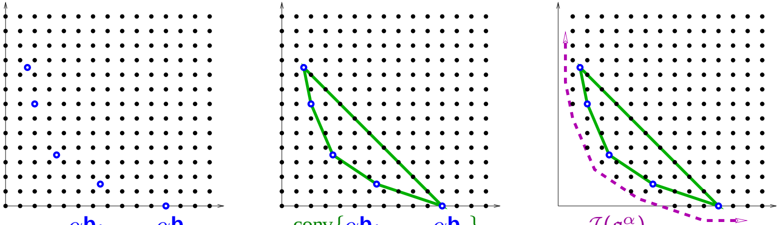

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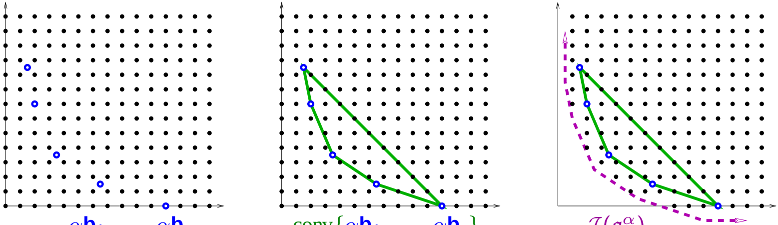

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$$\mathcal{J}((\mathfrak{a}_1 + \cdots + \mathfrak{a}_r)^\alpha) = \sum_{|\lambda| = \alpha} \mathcal{J}(\langle \mathbf{x}^{\lambda_1 \mathbf{b}_1} \cdots \mathbf{x}_r^{\lambda_r \mathbf{b}_r} \rangle) = \sum_{|\lambda| = \alpha} \mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r}).$$

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Cellular resolutions

Theorem 1 [Jow-M, 2007]. *There is a resolution*

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by sheaves $\mathcal{J}_i$ that are finite direct sums of multiplier ideals of the form $\mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r})$ for various nonnegative $\lambda \in \mathbb{R}^r$ with $\sum_{i=1}^r \lambda_i = \alpha$. Every distinct ideal sheaf of that form appears as a summand of $\mathcal{J}_0$.

Corollary [Takagi]. $\mathcal{J}((\mathfrak{a}_1 + \cdots + \mathfrak{a}_r)^\alpha) = \sum_{|\lambda|=\alpha} \mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r})$.

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to be a direct sum indexed by the set Δ_i of i-dimensional faces $\sigma \in \Delta$, with the differential of $\mathcal{J}_\bullet$ induced by natural maps between ideal sheaves, using the signs from the boundary maps of Δ. If $\lambda \in \Delta_0$ is a vertex, then $\mathcal{J}_\lambda = \mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r})$.

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Corollary [Takagi]. $\mathcal{J}((\mathfrak{a}_1 + \cdots + \mathfrak{a}_r)^\alpha) = \sum_{|\lambda|=\alpha} \mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r})$.

Theorem 2 [Jow-M, 2007]. *There is a triangulation Δ of the simplex $\{\lambda \in \mathbb{R}^r \mid \sum_{i=1}^r \lambda_i = \alpha$ and $\lambda \geq 0\}$ such that we can choose*

$$\mathcal{J}_i = \bigoplus_{\sigma \in \Delta_i} \mathcal{J}_\sigma$$

to be a direct sum indexed by the set Δ_i of i-dimensional faces $\sigma \in \Delta$, with the differential of $\mathcal{J}_\bullet$ induced by natural maps between ideal sheaves, using the signs from the boundary maps of Δ.
If $\lambda \in \Delta_0$ is a vertex, then $\mathcal{J}_\lambda = \mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r})$.

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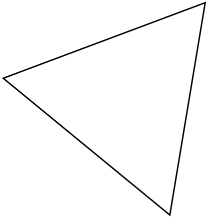

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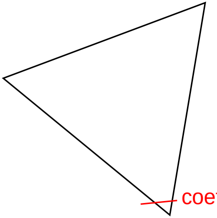

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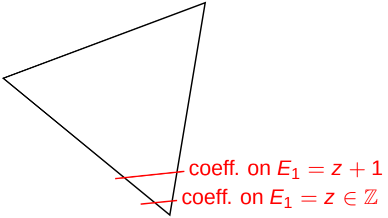

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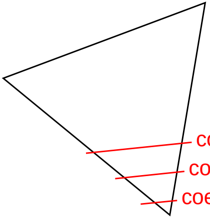

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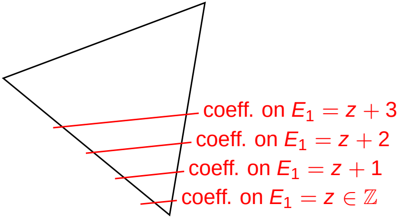

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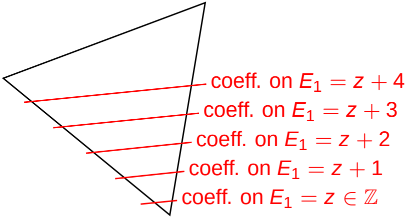

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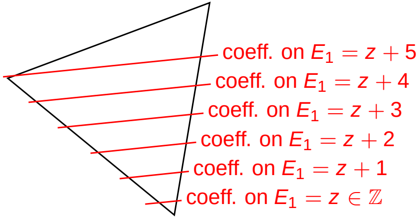

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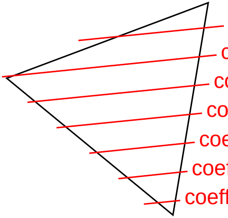

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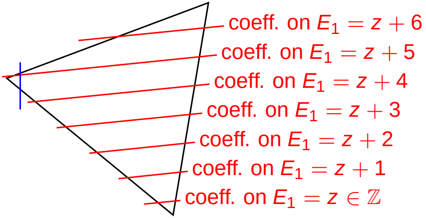

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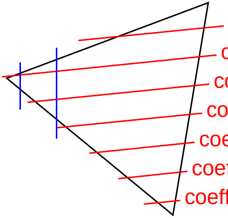

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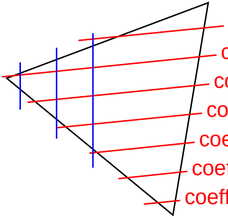

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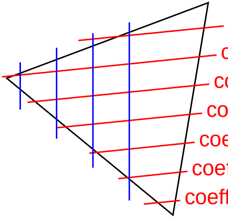

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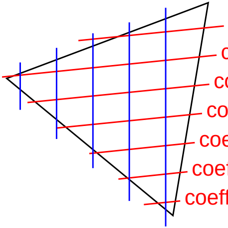

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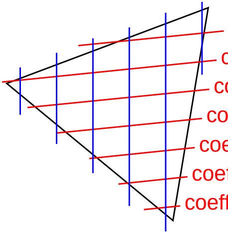

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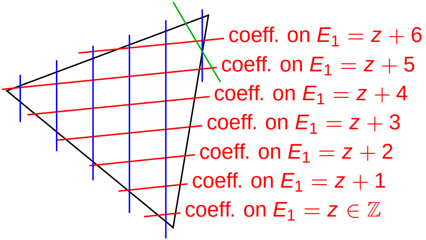

Set Δ = first barycentric subdivision of this polyhedral complex.

Theorem [Jow-M, 2007]. *For $\sigma \in \Delta$ set $m_\sigma = x_1^{\lfloor E_1(\lambda) \rfloor} \cdots x_n^{\lfloor E_n(\lambda) \rfloor}$, where λ is the barycenter of σ. Then Δ supports a cellular free resolution of $\langle m_\sigma \mid \sigma \in \Delta_0 \rangle$ over $\mathbb{C}[x_1, \ldots, x_n]$.*

This, locally on X', plus multiplier ideal hocus-pocus $\Rightarrow$ Thms 1 & 2.

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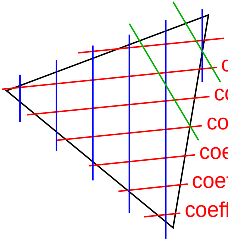

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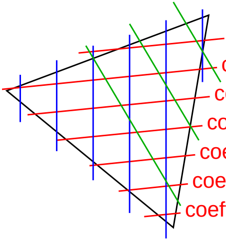

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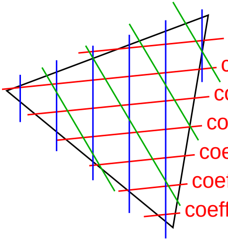

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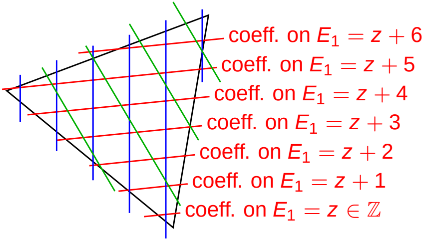

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