MULTIPLIER IDEALS OF SUMS VIA CELLULAR RESOLUTIONS

SHIN-YAO JOW AND EZRA MILLER

Abstract. Fix nonzero ideal sheaves $\mathfrak{a}_1, \ldots, \mathfrak{a}_r$ and $\mathfrak{b}$ on a normal $\mathbb{Q}$-Gorenstein complex variety X. For any positive real numbers α and β, we construct a resolution of the multiplier ideal $\mathcal{J}((\mathfrak{a}_1 + \cdots + \mathfrak{a}_r)^\alpha \mathfrak{b}^\beta)$ by sheaves that are direct sums of multiplier ideals $\mathcal{J}(\mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r} \mathfrak{b}^\beta)$ for various $\lambda \in \mathbb{R}^r_{\geq 0}$ satisfying $\sum_{i=1}^r \lambda_i = \alpha$. The resolution is cellular, in the sense that its boundary maps are encoded by the algebraic chain complex of a regular CW-complex. The CW-complex is naturally expressed as a triangulation Δ of the simplex of nonnegative real vectors $\lambda \in \mathbb{R}^r$ with $\sum_{i=1}^r \lambda_i = \alpha$. The acyclicity of our resolution reduces to that of a cellular free resolution, supported on Δ, of a related monomial ideal. Our resolution implies the multiplier ideal sum formula

$$\mathcal{J}(X, (\mathfrak{a}_1 + \cdots + \mathfrak{a}_r)^\alpha \mathfrak{b}^\beta) = \sum_{\lambda_1 + \cdots + \lambda_r = \alpha} \mathcal{J}(X, \mathfrak{a}_1^{\lambda_1} \cdots \mathfrak{a}_r^{\lambda_r} \mathfrak{b}^\beta),$$

generalizing Takagi's formula for two summands [Tak05], and recovering Howald's multiplier ideal formula for monomial ideals [How01] as a special case. Our resolution also yields a new exactness proof for the *Skoda complex* [Laz04, Section 9.6.C].

References

[BS98] Dave Bayer and Bernd Sturmfels, *Cellular resolutions of monomial modules*, J. Reine Angew. Math. **502** (1998), 123–140.

[Bjö84] Anders Björner, *Topological methods*, Handbook of combinatorics, Vol. 1, 2, 1819–1872, Elsevier, Amsterdam, 1995.

[DEL00] Jean-Pierre Demailly, Lawrence Ein and Robert Lazarsfeld, *A subadditivity property of multiplier ideals*, Michigan Math. J. **48** (2000), 137–156.

[How01] Jason Howald, *Multiplier ideals of monomial ideals*, Trans. Amer. Math. Soc. **353** (2001), no. 7, 2665–2671 (electronic).

[Laz04] Robert Lazarsfeld, *Positivity in Algebraic Geometry I–II*, Ergeb. Math. Grenzgeb., vols. **48**–**49**, Berlin: Springer, 2004.

[Mil00] Ezra Miller, *The Alexander duality functors and local duality with monomial support*, J. Algebra **231** (2000), 180–234.

[MS05] Ezra Miller and Bernd Sturmfels, *Combinatorial commutative algebra*, Graduate Texts in Mathematics **227**, New York: Springer, 2005.

[Mun84] James R. Munkres, *Elements of algebraic topology*, Addison–Wesley, Menlo Park, CA, 1984.

[Mus02] Mircea Mustaţă, *The multiplier ideals of a sum of ideals*, Trans. Amer. Math. Soc. **354** (2002), no. 1, 205–217.

[Tak05] Shunsuke Takagi, *Formulas for multiplier ideals on singular varieties*, Amer. J. Math. **128** (2006), 1345–1362. `arXiv:math.AG/0410612`

[Zie95] Günter Ziegler, *Lectures on polytopes*, Graduate Texts in Mathematics Vol. 152, Springer–Verlag, New York, 1995.

Date: 10 April 2007.

Key words and phrases. Multiplier ideal, summation formula, cellular resolution, monomial ideal, homology-manifold-with-boundary.