

RECIPROCAL DOMAINS AND COHEN–MACAULAY d -COMPLEXES IN \mathbb{R}^d

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Dedicated to Richard P. Stanley on the occasion of his 60th birthday

ABSTRACT. We extend a reciprocity theorem of Stanley about enumeration of integer points in polyhedral cones when one exchanges strict and weak inequalities. The proof highlights the roles played by Cohen–Macaulayness and canonical modules. The extension raises the issue of whether a Cohen–Macaulay complex of dimension d embedded piecewise-linearly in \mathbb{R}^d is necessarily a d -ball. This is observed to be true for $d \leq 3$, but false for $d = 4$.

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