to fill in more faces, or is this a general phenomenon? What choices did we make (implicitly or otherwise) that made things come out right? To understand the question better, look what happens to the last diagram in Example 7.10 if we choose the $\mathbb{Z}^n$-graded lift $(1, 0, 1)$ for the interior edge instead of $(0, 3, 0)$.

Solution B.4.7 See Example 7.17; we do not give details here.

References

[1] G. Agnarsson. The number of outside corners of monomial ideals. *J. Pure Appl. Algebra*, 117/118:3–21, 1997. Algorithms for algebra (Eindhoven, 1996).

[2] D. Bayer, I. Peeva, and B. Sturmfels. Monomial resolutions. *Math. Res. Letters*, 5:31–46, 1998.

[3] D. Bayer, S. Popescu, and B. Sturmfels. Syzygies of unimodular lattice ideals. Preprint `math.AG/9912247`, 1999.

[4] D. Bayer and B. Sturmfels. Cellular resolutions of monomial modules. *J. Reine Angew. Math.*, 502:123–140, 1998.

[5] A. M. Bigatti. Upper bounds for the Betti numbers of a given Hilbert function. *Comm. Algebra*, 21(7):2317–2334, 1993.

[6] L. J. Billera and A. Sarangarajan. The combinatorics of permutation polytopes. In *Formal power series and algebraic combinatorics (New Brunswick, NJ, 1994)*, pages 1–23. Amer. Math. Soc., Providence, RI, 1996.

[7] W. Bruns and J. Herzog. *Cohen-Macaulay rings*, volume 39 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1993.

[8] A. Capani, G. Niesi, and L. Robbiano. CoCoA, a system for doing computations in commutative algebra. Available by anonymous ftp at `cocoa.dima.unige.it`.

[9] D. Cox. The homogeneous coordinate ring of a toric variety. *J. Algebraic Geom.*, 4:17–50, 1995.

[10] D. Cox, J. Little, and D. O'Shea. *Ideals, Varieties, and Algorithms*. Undergraduate Texts in Mathematics. Springer-Verlag, New York, second edition, 1997. An introduction to computational algebraic geometry and commutative algebra.

[11] J. A. Eagon and V. Reiner. Resolutions of Stanley-Reisner rings and Alexander duality. *J. Pure and Appl. Algebra*, 130:265–275, 1998.

[12] D. Eisenbud. *Commutative algebra*, volume 150 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1995. With a view toward algebraic geometry.

[13] D. Eisenbud, M. Mustata, and M. Stillman. Cohomology of sheaves on toric varieties. Preprint, 1998.

[14] S. Eliahou and M. Kervaire. Minimal resolutions of some monomial ideals. *J. Algebra*, 129(1):1–25, 1990.

[15] R. Gebauer and H. M. Möller. On an installation of Buchberger's algorithm. *J. Symbolic Comput.*, 6(2-3):275–286, 1988. Computational aspects of commutative algebra.

[16] S. Goto and K. Watanabe. On graded rings, II ($\mathbb{Z}^n$-graded rings). *Tokyo J. Math.*, 1(2):237–261, 1978.

[17] H.-G. Gräbe. The canonical module of a Stanley-Reisner ring. *J. Algebra*, 86:272–281, 1984.

[18] D. Grayson and M. Stillman. *Macaulay 2.* Available by at the website `http://www.math.uiuc.edu/Macaulay2/`. A software system for algebraic geometry and commutative algebra.

[19] M. Hochster. Cohen-Macaulay rings, combinatorics, and simplicial complexes. In B. R. McDonald and R. Morris, editors, *Ring Theory II*, number 26 in Lect. Notes in Pure and Appl. Math., pages 171–223. Dekker, New York, 1977.

[20] S. Hoşten and W. Morris, Jr. The order dimension of the complete graph. *Discrete Math.*, 201:133–139, 1999.

[21] H. A. Hulett. Maximum Betti numbers of homogeneous ideals with a given Hilbert function. *Comm. Algebra*, 21(7):2335–2350, 1993.

[22] F. S. Macaulay. Some properties of enumeration in the theory of modular systems. *Proc. London Math. Soc.*, 26:531–555, 1927.

[23] E. Miller. Alexander duality for monomial ideals and their resolutions. Preprint (`math.AG/9812095`), 1998.

[24] E. Miller. The Alexander duality functors and local duality with monomial support. Submitted; available at `http://www.math.berkeley.edu/~enmiller`, 1999.

[25] E. Miller and B. Sturmfels. Monomial ideals and planar graphs. In M. Fossorier, H. Imai, S. Lin, and A. Poli, editors, *Applied Algebra, Algebraic Algorithms and Error-Correcting Codes*, number 1719 in Springer Lecture Notes in Computer Science, pages 19–28. Springer Verlag, 1999. Proceedings of AAECC-13 (Honolulu, Nov. 1999).

[26] E. Miller, B. Sturmfels, and K. Yanagawa. Generic and cogneric monomial ideals. *J. Symbolic Comp.*, To appear.

[27] E. N. Miller. *Resolutions and Duality for Monomial Ideals.* PhD thesis, University of California at Berkeley, 2000.

[28] M. Mustaţă. Vanishing theorems on toric varieties. Preprint `math.AG/0001142`, 1999.

[29] M. Mustaţă. Local cohomology at monomial ideals. Preprint `math.AG/0001153`, 2000.

[30] I. Peeva and B. Sturmfels. Generic lattice ideals. *J. Amer. Math. Soc.*, 11:363–373, 1998.

[31] M. Saito, B. Sturmfels, and N. Takayama. *Gröbner Deformations of Hypergeometric Differential Equations*, volume 6 of *Algorithms and Computation in Mathematics.* Springer-Verlag, Berlin, 1999.

[32] R. P. Stanley. *Combinatorics and Commutative Algebra*, volume 41 of *Progress in Mathematics.* Birkhäuser, second edition, 1996.

[33] B. Sturmfels. *Gröbner Bases and Convex Polytopes*, volume 8 of *AMS University Lecture Series.* American Mathematical Society, Providence RI, 1995.

[34] B. Sturmfels. The co-Scarf resolution. In D. Eisenbud, editor, *Commutative Algebra, Algebraic Geometry, and Computational Methods*, pages 315–320. Springer Verlag, Singapore, 1999.

[35] B. Sturmfels, N. V. Trung, and W. Vogel. Bounds on degrees of projective schemes. *Math. Ann.*, 302:417–432, 1995.

[36] D. Taylor. *Ideals Generated By Monomials in an R-sequence.* PhD thesis, University of Chicago, 1961.

[37] N. Terai. Generalization of Eagon-Reiner theorem and h-vectors of graded rings. Preprint, 1997.

[38] N. Terai. Local cohomology modules with respect to monomial ideals. Preprint, 1999.

[39] G. M. Ziegler. *Lectures on Polytopes.* Graduate Texts in Mathematics. Springer-Verlag, New York, 1995.