Monomial ideals and planar graphs

Gröbner basis theory reduces questions about systems of polynomial equations to the combinatorial study of monomial ideals, or staircases. This article gives an elementary introduction to current research in this area. After reviewing the bivariate case, a new correspondence is established between planar graphs and minimal resolutions of monomial ideals in three variables. A brief guide is given to the literature on complexity issues and monomial ideals in four or more variables.