ALGORITHMS FOR GRADED INJECTIVE RESOLUTIONS AND LOCAL COHOMOLOGY OVER SEMIGROUP RINGS

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ABSTRACT. Let Q be an affine semigroup generating $\mathbb{Z}^d$, and fix a finitely generated $\mathbb{Z}^d$-graded module M over the semigroup algebra $\Bbbk[Q]$ for a field $\Bbbk$. We provide an algorithm to compute a minimal $\mathbb{Z}^d$-graded injective resolution of M up to any desired cohomological degree. As an application, we derive an algorithm computing the local cohomology modules $H^i_I(M)$ supported on any monomial (that is, $\mathbb{Z}^d$-graded) ideal I. Since these local cohomology modules are neither finitely generated nor finitely cogenerated, part of this task is defining a finite data structure to encode them.

CONTENTS

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Date: 15 September 2003.

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