Graded Greenlees-May Duality and the Čech Hull

The duality theorem of Greenlees and May relating local cohomology with support on an ideal I and the left derived functors of I-adic completion [GM92] holds for rather general ideals in commutative rings. Here, simple formulas are provided for both local cohomology and derived functors of $\mathbb{Z}^n$-graded completion, when I is a monomial ideal in the $\mathbb{Z}^n$-graded polynomial ring $k[x_1, \ldots, x_n]$. Greenlees-May duality for this case is a consequence. A key construction is the combinatorially defined Čech hull operation on $\mathbb{Z}^n$-graded modules [Mil98, Mil00, Yan00]. A simple self-contained proof of GM duality in the derived category is presented for arbitrarily graded noetherian rings, using methods motivated by the Čech hull.

References

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