Math 627 Homework #2, Fall 2022
Instructor: Ezra Miller

Solutions by: ...your name...
Collaborators: ...list those with whom you worked on this assignment...

Due: Tuesday 27 September 2022

Reading assignments in [Vakil]
- by Tuesday 20 September: Chapter 3; most of this should be review
- by Thursday 22 September: §4.1–§4.4
- by Tuesday 27 September: §4.5, Chapter 5 (much of this should be review)
- by Thursday 29 September: §11.1–§11.3, §12.1–§12.3 and §12.5; this plus Chapter 5 is a lot of material, but most should be review (skip items mentioning scheme morphisms)

Exercises: In [Vakil], exercises have labels C.S.N, for “Chapter C, Section S, Exercise N”, where C, S \( \in \mathbb{Z}_+ \) and N \( \in A, \ldots, Z \). Exercises marked “[essential]” are essential.

2.6.J
2.7.C
2.7.G (a)
  (b) [essential]
  (c) [essential]
2.5.D [essential]
3.2.Q [essential]
3.4.C (a)
  (b)
  (c)
3.4.H [essential]
3.6.L
3.6.F (a)
  (b)
3.6.T
3.7.F [essential]
3.6.Q
3.7.E
4.1.A
4.3.F [essential]
13.1.A
13.1.C [essential]
13.1.E [essential]

Non-book Exercise

1. Fix a coherent sheaf \( \mathcal{F} \) on a scheme \((X, \mathcal{O}_X)\). Prove that the set of points \( p \in X \) where the fiber \( \mathcal{F}(p) \) has dimension at least \( r \) is closed in \( X \), for each \( r \geq 0 \). Hint: what condition on an \( m \times n \) matrix with entries in a field guarantees that it has rank at most \( n - r \)? [You need only what we did in class concerning coherent sheaves for this.]

References