Math 221 Homework for Section 4.3:
additional problems on change of basis

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1. Let $V$ be an inner product space of dimension 7 with a subspace $W$ of dimension 2. Fix linearly independent $w_1, w_2 \in W$ and linearly independent $v_3, v_4, v_5, v_6, v_7 \in V$ all orthogonal to $W$. Then $B = w_1, w_2, v_3, v_4, v_5, v_6, v_7$ is a basis for $V$ (why?). Use the general formula

$$T[x_1 \cdots x_n] = [x_1 \cdots x_n] [T]_B$$

with $T = \text{proj}_W$ to find the $7 \times 7$ matrix $[\text{proj}_W]_B$ for the orthogonal projection of $V$ onto $W$ with respect to $B$. Is anything simpler if $w_1$ and $w_2$ are orthogonal or orthonormal? Hint: which of these basis vectors does $\text{proj}_W$ fix? What happens to the others?

2. In Question 1, suppose $V = \mathbb{R}^7$ and

$$w_1 = \begin{bmatrix} 0 \\ 1 \\ -2 \\ 0 \\ 0 \\ 3 \\ -1 \end{bmatrix} \quad \text{and} \quad w_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$ 

Let $A = [\text{proj}_W]_E$ be the matrix of the projection onto $W$ with respect to the standard basis. Express $A$ as a product of matrices and their inverses; do not attempt to invert or multiply the matrices.

3. The polynomial $f(t) = 6 - 2t^2 + t^3 - \pi t^4$ can be expressed as a matrix product

$$f(t) = \begin{bmatrix} 1 & t^2 & t^3 & t^4 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ -2 \\ \frac{1}{\pi} \end{bmatrix}.$$ 

This polynomial $f(t)$ is a linear combination of the polynomials in the row vector $[1+t+t^2 \ t+t^2+t^3 \ t^2+t^3+t^4 \ t^3+t^4 \ t^4]$. The coefficients in this linear combination are the entries of a column vector of size 5. Express that column as the product of a matrix and the column vector displayed above; do not attempt to invert or multiply any matrices (unless you’d like to check your answers!). Hint: additional problem 3 from Section 4.4.