1. Math 221 221.02 Fall 2021

Linear Algebra

Tue / Thu 12:00 – 13:15

Physics 119

Text: Shifrin & Adams ← all exercises taken from here

Office Hours: Mon 16:15 – 17:15
Thu 13:15 – 14:30 right after class } outside if possible Physics 209 if not

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Safety:
- masks, hand wash/sanitize — politely point out noncompliance — even me!
- distance if possible
- know where the exits are — from the room and the building

Policies
- covered on Thu ⇒ fair game for HW or exam Tue
- collaboration / academic honesty
  - Yes on HW
  - No on exams — I have
    - brought numerous cases to the Office of Student Conduct
    - never lost

Index cards

1. Ezra Miller
2. he / him
3. 42nd grade
4. Major or potential major: Math, Music
5. What you hope to get out of this course

   students who know linear algebra, especially the right way to think about rank

6. The most important thing you've learned about how you learn

   not to take notes!

7. Hobbies: frisbee, gardening, photography, beer
8. Something unique about yourself

   hold breath for 4 minutes
   screws in right hand
   told by doctor in hospital I was going to die of rabies
   so radioactive I set off a Geiger counter from across room
   remarkable bike accident without injury

X "I'm from MA"

✓ "I'm from HI —

but I'm allergic to pineapple!"

What this course covers

1. things that are straight or flat "linear"
2. systems of linear equations and their solutions "algebra"
   $y=x+2$ not $y=x^2$
3. ways to manipulate these while preserving straightness, flatness, or linearity
4. how to · write down
   · understand (e.g. decompose into simpler pieces)
   manipulations ("operations") of this sort
   includes
5. solving linear systems
6. applications: so many phenomena behave linearly or — as you've seen in Calculus — approximately so $\Rightarrow$ linear algebra is the foundation of modern math & stat

This is a serious mathematical, abstract introduction.
Proofs are essential, both in class and in written assignments, including exams.
If you are taking this class purely to use linear algebra, then consider Math 212 or 218; we'll help you find a section
Course is geared toward (potential) math majors and minors.

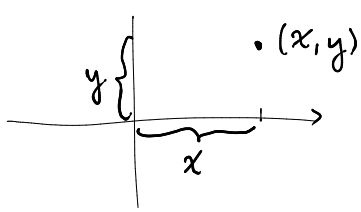
Vectors §1.1 – 1.2

Ask about any unfamiliar symbols!

$\mathbb{R}$ = real numbers $x \in \mathbb{R}$ is a partition of the rational #s into a left half and a right half

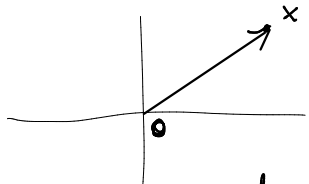
$\mathbb{R}^n = \{(x_1, \dots, x_n) \mid x_1 \in \mathbb{R}, \dots, x_n \in \mathbb{R}\}$ ← This is a sentence!

E.g. $\mathbb{R}^2 =$



Def: A vector in $\mathbb{R}^n$ is a point $X = (x_1, \dots, x_n) \in \mathbb{R}^n$.

to visualize / represent / draw a vector:

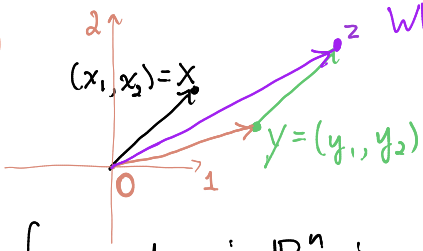


an arrow with tail at O, head at X

Convention: same vector if arrow is moved around.

Q. If you see two arrows, how do you tell if they represent the same vector?

A. (poll class)



What must the coordinates of this point be?

0 to x = y to z (do "rise and run")

$\Leftrightarrow$ $z_1 - y_1 = x_1 - 0$ $\quad z_2 - y_2 = x_2 - 0$

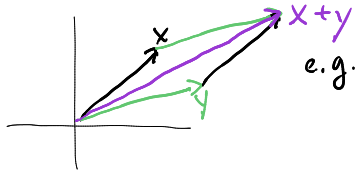
$z_1 = x_1 + y_1$ $\quad z_2 = x_2 + y_2$

$\Rightarrow$ $z = x + y$.

<u>Def</u>: The <u>sum</u> of x and y in $\mathbb{R}^n$ is

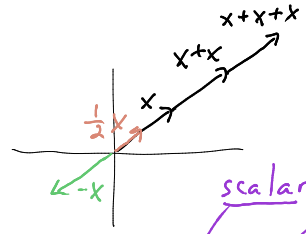
$$x + y = (x_1 + y_1, \ldots, x_n + y_n).$$

Better picture:



e.g. $x = (5,4)$, $y = (7,2)$ $\Rightarrow$ $x + y = (5+7, 4+2) = (12, 6)$

Q. What does $x + x$ look like? $\quad$ A.



$x + x + x$?

What would $\frac{1}{2}x$ be?

<u>Def</u>: For $x \in \mathbb{R}^n$ and $c \in \mathbb{R}$, the <u>scalar multiple</u> of x by c is $cx = (cx_1, \ldots, cx_n) \in \mathbb{R}^n$.

scalar

set of all scalars

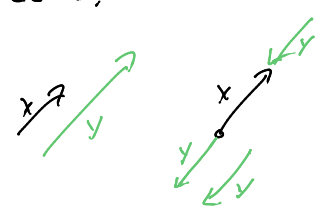
"change of scale"

really "set"

space of vectors ("vector space")

<u>Def</u>: Nonzero x and y are <u>parallel</u> if $y = cx$ for some $c \in \mathbb{R}$.

looks like x, y collinear, but now draw various reps:



Q. What is a line in $\mathbb{R}^n$?

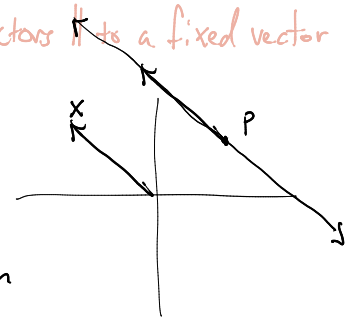
A. through 0: $\{cx \mid c \in \mathbb{R}\}$ for some $x \in \mathbb{R}^n$ $\quad$ the set of all vectors $\parallel$ to a fixed vector

$\overset{\text{def}}{=} \operatorname{span}(x)$.

through $p = (p_1, \ldots, p_n)$: $\{p + cx \mid c \in \mathbb{R}\}$ for some $x \in \mathbb{R}^n$

parameter

point $\quad$ vector

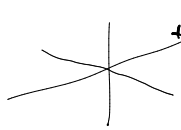


<u>parametric</u> description of a line; c = parameter

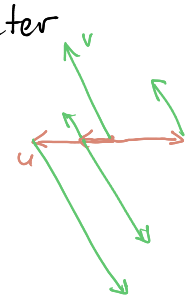
in $\mathbb{R}^2$, <u>implicit</u> description: $z_2 = mz_1 + b$

Q. What is a plane in $\mathbb{R}^n$?

through 0:



3 chop sticks + 2 pencils $\quad$ forget the axes $\rightsquigarrow$



$$\{su + tv \mid s \in \mathbb{R},\ t \in \mathbb{R}\} = \operatorname{span}(u, v)$$

through p: $p + \operatorname{span}(u, v)$