EXAM 3

Math 219, 2024 Spring, Clark Bray.

Name:_____ Section:_____ Student ID:_____

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not remove the staple, tear pages out of the staple, or tamper with the exam packet in any way. Do not write anything near the staple – this may be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams (only!), but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the physical page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

You may use a straight edge to assist in your drawings, but ONLY if there is zero mathematical content on the item.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

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1. *(20 pts)* The curve C is parametrized by $\vec{x}(t) = (\cos t, \sin t, t)$, with $t \in [0, \pi]$, and we have $\vec{F}(x, y, z) = (z - y,\ -x - 5y^4,\ x)$.

   (a) Compute $\int_C x\, ds$.

   (b) Compute $\int_C \vec{F} \cdot d\vec{x}$.

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2. *(20 pts)* H is the solid regular hexagon in the xy-plane above the x-axis with one edge as the line segment from $(0,0)$ to $(1,0)$, and we have the vector field $\vec{F}(x,y) = (3x - 4y, 5x - 6y)$.

   (a) Compute the circulation of $\vec{F}$ *clockwise* around the perimeter of H, <u>*without*</u> parametrizing.

   (b) Compute the flux of $\vec{F}$ *inward* through the perimeter of H, <u>*without*</u> parametrizing.

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3. *(20 pts)* The curve T is an equilateral triangle in the plane with equation $11x - y + 5z = 49$, with vertices at $(2, 3, 6)$, $(3, -6, 2)$, $(6, 2, -3)$, oriented in the direction of this listing of the vertices. Compute the circulation along T of the vector field $\vec{F}(x, y, z) = (x + 2y + 3z, 4x + 5y + 6z, 7x + 8y + 9z)$ *without* parametrizing the edges of T.

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4. *(20 pts)* S is the inward-oriented surface of the solid $[0,1] \times [2,5] \times [1,3]$. Compute the flux through S of the vector field $\vec{F}(x,y,z) = (y^3 - e^{z^2}, 4y + xz - z^2 e^x, x^3 y^2 + z^2)$.

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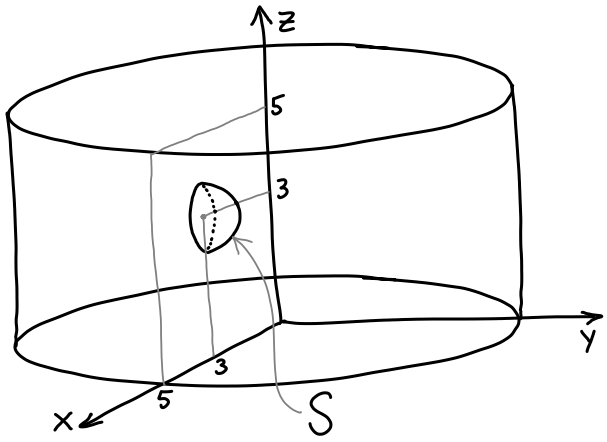
5. *(20 pts)* The water in a cylindrical pot (base of radius 5 sitting on the xy-plane, height of 5) is stirred such that the water is moving with velocity $\vec{F}(x, y, z) = (-y, x, 0)/\sqrt{x^2 + y^2}$. The surface S, the part of $y = 1 - (x-3)^2 - (z-3)^2$ with $y \geq 0$, describes the shape of a strainer. Compute the flux $\iint_S \vec{F} \cdot d\vec{S}$, giving the flow rate of water through the strainer in the positive y-direction.



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