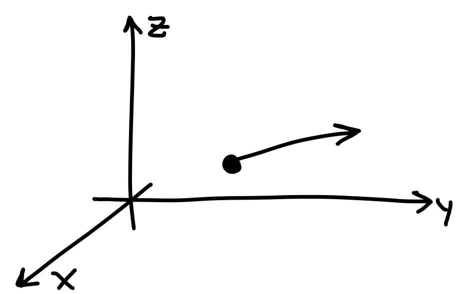
Introduction – Why is multivariable calculus important?

We already know single variable calculus is important:

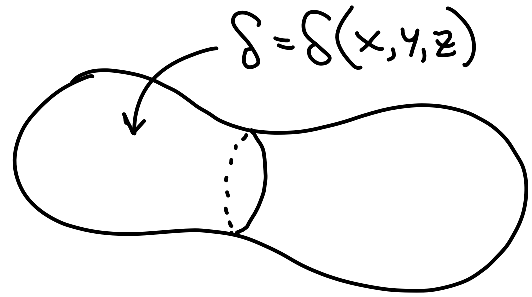
- to understand how things change (derivatives!)
- to compute accumulating quantities (integrals!)

But the world is 3-d, not 1-d!

Ex:) If temperature depends on x, y, and z and we move at an angle in space, what is dT/dt?



Ex:) If density depends on x, y, and z, how do we compute the mass in a region?



There are applications in many fields of study.

Ex:) Economics: Say P, Q, i, K, L are related by

$$LP^3 - Pe^K + \ln(1+i) - Qe^P + L^3 = 0$$

How does $\frac{dP}{dt}$ depend on $\frac{dL}{dt}$, $\frac{dK}{dt}$, $\frac{dQ}{dt}$, $\frac{di}{dt}$?

NB, there are 5+1 variables here! It is not enough to limit to 3-d!

Ex:) Physics, Engineering: Much of E&M is described by

$$\iiint_R \frac{\rho}{\epsilon_0}\, dV = \iint_{\partial R} \vec{E}\cdot d\vec{S} \qquad \nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$0 = \iint_{\partial R} \vec{B}\cdot d\vec{S} \qquad \nabla\cdot\vec{B} = 0$$

$$-\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = \int_{\partial S} \vec{E}\cdot d\vec{x} \qquad \nabla\times\vec{E} = -\frac{d\vec{B}}{dt}$$

$$\mu_0 I = \int_{\partial S} \vec{B}\cdot d\vec{x} \qquad \nabla\times\vec{B} = \mu_0\vec{J}$$

What does this mean? How can we best understand these statements, and what they fundamentally mean about these quantities?

Ex:) Chemistry, physics: Molecular bonds come from the behavior of electrons, which obey Schrödinger's equation:

$$i\hbar \frac{d}{dt}\psi = -\Delta\psi + V\psi$$

What does this mean? How do we solve it?

... and there are <u>many</u> more such applications.

Multivariable calculus is the calculus of the real world!