1. Which of the following subspaces are open in the given space?

   (a) \{the \ x\text{-axis}\} \subset \mathbb{R}^2
   
   (b) \{(x, y)|x + y \leq 3\} \subset \mathbb{R}^2
   
   (c) \{S^2 - \{\text{the equator}\}\} \subset S^2

2. For these two problems, let \{a_1, \ldots, a_n\} be arbitrary elements of \(G\).

   (a) Show that

   \[N = \left\{ \text{the collection of all products of powers of conjugates of } \{a_1, \ldots, a_n\} \right\}\]

   is a subgroup of \(G\) by

   i. showing that \(N\) contains the identity of \(G\);
   
   ii. showing that the product of two elements of \(N\) is also an element of \(N\);
   
   iii. showing that the inverse of an element of \(N\) is an element of \(N\)

   (b) Show that \(N\) is a normal subgroup of \(G\) by verifying that every conjugate of an element of \(N\) is also an element of \(N\).

3. (a) Following the same sort of procedure used in class, calculate the fundamental group of the Klein bottle using the Seifert-Van Kampen theorem.

   (b) Given the above conclusion, show that \(\pi_1(K)\) is not abelian by finding two elements of \(\pi_1(K)\) that do not commute.

4. Find the fundamental group of the projective plane using the Seifert-Van Kampen theorem.

5. Show that the sphere is simply connected by considering it as the union of the two sets \(A = S^2 - \text{the north pole}\) and \(B = S^2 - \text{the south pole}\).