EXAM II
Math 41, Fall 2001.
You have 2 hours.
No notes, no books, no calculators.
YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT
Good luck!

Name _______________________________________
ID number__________________

1. __________ ( /20 points) “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

2. __________ ( /30 points)

Signature: ____________________________

3. __________ ( /20 points) Circle your TA’s name:

4. __________ ( /30 points) Ted Hwa (2 and 7)

5. __________ ( /20 points) Yu Yan (3 and 8)

6. __________ ( /30 points) Brett Parker (4 and 9)

Bonus __________ ( /15 points) Ryan Vinroot (5 and 10)

Circle your section meeting time:

Total __________ ( /150 points) Alex Meadows (A03)

11:00am 1:15pm 7pm
1. Find the derivatives of the following functions:

a) 

\[ f(x) = \ln(1 + e^{\sin x}) \]

b) 

\[ g(x) = \sqrt{1 + x^x} \quad \text{(assume } x > 0) \]
2. (a) Find the derivative of \( xe^x \).

(b) Find the derivative of \( (\sin x)e^x \).

(c) Let \( f \) be differentiable; find (in terms of \( f \) and \( f' \)) the derivative of

\[ f(x)e^x \]
3. (a) The “Folium of Descartes” is the set of solutions to the equation

\[ x^3 + y^3 = 3axy \]

where \(a\) is a constant. Find \( \frac{dy}{dx} \) as a function of \(x\) and \(y\).

(b) Find \( \frac{dy}{dx} \) as a function of \(x\) and \(y\) for the “Kampyle of Eudoxus”, which is the set of solutions to

\[ a^2x^4 = b^4(x^2 + y^2) \]

where \(a\) and \(b\) are constants.
4. (a) Find the first, second, and third derivatives of

\[ f(x) = \ln(1 + x) \]

(b) Noticing a pattern in the answers from part (a), what is the \( n \)th derivative of \( f(x) \)?
(c) Using the results from parts (a) and (b), write down the Taylor series (with $a = 0$) for $f(x) = \ln(1 + x)$. Write your answer in the form

$$T(x) = (0\text{th term}) + (1\text{st term}) + (2\text{nd term}) + \ldots + (n\text{th term}) + \ldots$$

(d) Use the second order Taylor polynomial to estimate $\ln(1.001)$. 
5. Find the absolute maximum value of the function

\[ f(x) = x - x^3 \]

defined on the interval \([-3, 2]\). Explain all of your reasoning.
6. Evaluate the following limits using L’Hôpital’s Rule:

(a) \[ \lim_{x \to \infty} x^{(1/x)} \]

(b) \[ \lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} \]
(c) Rewrite the limit from part (b) using the Taylor series for $\sin x$ (below), and use this as an alternative method to compute the limit.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$
**Bonus Question:**

We are given a tank full of water, which is evaporating out of the top at a rate that is proportional to the surface area of water that is exposed to the air.

However, we are not given any information about the actual shape of the tank.

We define $V(h)$ to be the volume of water in the tank when the depth of the water is $h$. We define $A(h)$ to be the exposed surface area when the depth of the water is $h$.

For this problem, you may assume also that the rate of change of $V(h)$ with respect to $h$ is $A(h)$.

Use the chain rule to show that the depth of the water decreases at a constant rate (with respect to time) as a result of the evaporation, **independent of the shape of the tank**.