(The problems below make use of “imaginary numbers”. All you need to know about imaginary numbers to work these problems is that $i$ is defined to be $\sqrt{-1}$; and thus that $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, ...)

1. Use the formula for the Taylor series, derived in class, to find the Taylor series (using $a = 0$) for these functions:
   
   (a) $e^x$
   (b) $\sin(x)$
   (c) $\cos(x)$

2. Prove the three identities below by showing that in each equation, both sides of the equation have the same Taylor series. (Hint: You can use the three series that you derived in the previous problem.)

   (a) $e^{ix} = \cos x + i \sin x$
   (b) $\cos ix = \frac{e^{-x} + e^x}{2}$
   (c) $\sin ix = \frac{e^{-x} - e^x}{2i}$

3. Show that in problem 2, equations (b) and (c) can be derived from (a) directly, without using any Taylor series.