1. a) Let $B$ be Sally’s balance. Then $\frac{dB}{dt}$ is the rate at which she receives interest minus the rate at which she withdraws money:

$$\frac{dB}{dt} = (.05)B - 1500.$$ 

The only equilibrium is found by setting $\frac{dB}{dt} = 0$ to get $B = 1500/(.05) = 30,000$. If her balance is slightly more than this (that is, if $B > 30,000$) then $\frac{dB}{dt}$ is positive, and so her balance increases. Similarly if her balance is less than 30,000 then $\frac{dB}{dt}$ is negative and so her balance decreases. Therefore the equilibrium is unstable.

b) Separating variables and integrating gives

$$\int \frac{dB}{B/20 - 1500} = \int dt$$

$$20 \ln |B/20 - 1500| = t + C$$

$$|B/20 - 1500| = C_0 e^{t/20}$$

We may remove the absolute value bars by replacing $C_0$ by $-C_0$ if necessary and we arrive at

$$B = 30,000 + 20C_0 e^{t/20}$$

From the initial condition $B(0) = 5,000$ we deduce $-25,000 = 20C_0$ giving

$$B = 30,000 - 25,000e^{t/20}.$$
To determine when Sally goes broke we set $B = 0$ and solve for $t$:

\[
30,000 = 25,000e^{t/20} \\
6/5 = e^{t/20} \\
20 \ln \frac{6}{5} = t
\]

2. When the water in the tank is down to height $h$, the surface of the exposed water is a disk of radius $r = hR/H$. Recall (or derive) that the volume of a cone of radius $r$ and height $h$ is $V = \frac{1}{3} \pi r^2 h$ and so the volume of the remaining water is

\[
V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi h^3 R^2 / H^2
\]

Differentiating with respect to $t$ gives

\[
\frac{dV}{dt} = \frac{\pi R^2}{H^2} h^2 \frac{dh}{dt}
\]

which we are told is proportional to $h$. If we let $k < 0$ denote the constant of proportionality we are led to the differential equation

\[
kh = \frac{\pi R^2}{H^2} h^2 \frac{dh}{dt}
\]

which we rewrite as

\[
\frac{dh}{dt} = \frac{kH^2}{\pi R^2} \cdot \frac{1}{h}
\]

Separating the variables and integrating gives the solution

\[
h = \sqrt{\frac{2kH^2}{\pi R^2}} t + C
\]

for some constant $C$. To determine $C$ we use the fact that $h(0) = H$ (i.e. the tank starts out full). Setting $t = 0$ above we see that $C = H^2$ so that our solution becomes

\[
h(t) = \sqrt{\frac{2kH^2}{\pi R^2}} t + H^2
\]

\[
= H \sqrt{\frac{2kt}{\pi R^2} + 1}
\]

To find the time for all of the water to drain out, set $h(t) = 0$ and solve for $t$:

\[
t = -\frac{\pi R^2}{2k}
\]